Modelling and receding-horizon control of maritime container terminals

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Abstract. The main objective of this paper consists in modelling, optimizing and controlling container transfer operations inside intermodal terminals. More specifically, maritime container terminals are here considered, involving three kinds of transportation modes, maritime, rail and road transport. Generally speaking, an intermodal port terminal can be seen as an open system of container flows with two interfaces, towards the hinterland and towards the sea, respectively. Moreover, inside a terminal, unloading operations of inbound containers, container storage and loading operations of outbound containers are performed.

A simple model for maritime container terminals is proposed in this paper. In the model, a set of queues represent the position of containers and their movements inside the terminal. The dynamic evolutions of these queues are described by discrete-time equations, where the state variables represent the queue lengths and the control variables take into account the utilization of terminal resources as load/unload handling rates. On the basis of the proposed model, an optimization problem is defined that consists in minimizing the transfer delays of containers in the terminal. The problem is stated as an optimal control problem whose solution is sought by adopting a receding-horizon strategy.
1. Introduction

The increasing development of container traffic and the growing competition between seaports has caused, in the last decades, the evolution of container terminals, regarding both the logistics management systems and the technical equipment. In particular, the efficiency of an intermodal container terminal is influenced by the ability to manage and coordinate transfer and storage activities performed by the handling systems used inside the terminal; as a matter of fact, an important success factor for a container terminal is the reduction of transhipment time, that is the time spent by ships in the port. In this sense, to ensure a high level of efficiency and coordination inside a terminal, the use of information and communication technology and automated control systems has played an important role for some years.

The improvement of the overall system productivity can be achieved by suitably modelling and optimizing the terminal resources and their utilization. In this context, two main research issues must be considered, the former relevant to the definition of suitable models for terminal operations and the latter concerning the use of such models to find a decision strategy for optimizing and controlling container transfers inside terminals.

Since optimization methods of logistic operations inside container terminals are considered an important issue, an ever increasing number of publications have appeared in the literature, in particular in the last decade. Two overview papers in the area of container terminal logistics are (Vis and de Koster, 2003) and (Steenken et al., 2004), which classify the main decision problems that regard container terminals, providing an overview of the relevant literature. Both these papers stress the fact that, until now, the main research works have been focused on the optimization of single processes inside the terminal, such as berth allocation, stowage planning, storage and transport optimization. Anyway,
an integrated approach considering all these problems and aimed at optimizing the whole terminal, is becoming really necessary.

In this sense, simulation models and analytical methods can be used, both being affected by advantages and drawbacks. A simulation model can represent all the features of a real container terminal and can be very useful to test different scenarios or to compare alternative solutions to be applied to the terminal. For instance, Kia et al. (Kia et al., 2002) investigate the role of computer simulation in evaluating the performance of a container terminal, Liu et al. (Liu et al., 2002) use future demand scenarios to design the characteristics of different terminals, developing a microscopic simulation model, while Gambardella et al. (Gambardella et al., 1998) show how operation research techniques can be used to generate resource allocation plans.

Actually, a major drawback of simulation approaches stands in the fact that the model development and, above all, its validation turn out to be cumbersome and time-consuming phases. On the other hand, analytical models generally require less time to be developed and applied, but they typically need suitable model assumptions and problem simplifications, possibly affecting their effectiveness. Van Hee et al. (Van Hee and Wijbrands, 1988; Van Hee et al., 1988) develop a decision support system for container terminal and port terminal planning, including several mathematical models. In Kozan (Kozan, 2000), a network model, representing the logistic structure of a container terminal, is developed with the objective of minimizing the total throughput time.

In this paper, which gathers and extends the results presented in (Alessandri et al., 2004a; Alessandri et al., 2004b), a different approach is followed that consists in using an “ad hoc” model which constitutes the basis for stating and solving a suitable optimization problem. Specifically, the proposed model is based on a set of queues; each queue represents a different container allocation stage in the area of
the terminal, where containers are stored, depending on the transport modality and routing. The arrival/departure flows of containers are regarded as exogenous random inputs, and the adopted cost function is a performance measure of the overall terminal, specifically related to the queue occupancy. The proposed model represents the dynamic behaviour of the overall system, for which several optimization problems relevant to different decisional levels can be designed. As a matter of fact, the defined framework can be adopted at a strategic (long–term) planning phase (as done in the present paper), in order to define such system parameters as the number of container handling resources in the different areas of the terminal. On the other hand, the same model can also be adopted at a tactical (medium–term) planning phase or even to determine an optimal control strategy to be applied to the system in an operation (short–term) planning phase. In all such cases, the optimization problem is faced in a receding-horizon scheme in which, at each time step, decisions are taken so as to minimize the cost function over a sliding horizon.

The paper is organized as follows. In Section II, our model for a maritime container terminal is described, analysing its discrete-time equations and the necessary constraints. In Section III, a receding-horizon scheme is considered for the solution of the optimization problem consisting in minimising the transfer delays of containers inside the terminal. Then, the effectiveness of the proposed control scheme is tested in a case study, as described in Section IV. Finally, some conclusions are given in Section V.
2. A model of the intermodal terminal operations

A general structure of maritime container terminals is depicted in Fig. 1. Different areas can be identified in the proposed structure; first of all, a quay is a part of the terminal devoted to handling operations with container ships. In the proposed model, specific attention has been devoted to the management of container ships by quay resources, since this turns out to be a crucial element of the overall terminal performance. In particular, it has been noted that several kinds of ships, characterized by quite different sizes (and, then, requiring different handling policies), typically reach a maritime container terminal. Without loss of generality, three kinds of container ships are assumed to be involved in the terminal operations. This seems to be a quite general configuration at least for Mediterranean terminals. As a matter of fact, there is no difficulty in varying the number of ship types to be considered in the model, by possibly reducing or increasing it. As regards the other transportation modes, gates for trucks and railway yards for trains are present in the terminal. Moreover, the transfer machines operating in the terminal are quay cranes, transfer cranes, transtainers, reach stackers and yard trucks.

As already mentioned, the overall container terminal is modelled by representing the different physical areas as buffers. Fig. 2 pictorially describes the proposed model, that is a discrete–time model in which a time discretization is made. In a generic time interval \( t \), the arrival rates of containers at the terminal by ship (divided in 3 different typologies), truck and train are given by quantities \( a_1(t), a_2(t), a_3(t), a_4(t) \) and \( a_5(t) \), respectively. Analogously, \( d_1(t), d_2(t), d_3(t), d_4(t) \) and \( d_5(t) \) model departure rates of containers from the same transport modes.

The arrival (departure) processes \( a_i(t), i = 1, \ldots, 5, t = 0, 1, \ldots \) \( (d_i(t), i = 1, \ldots, 5, t = 0, 1, \ldots) \) are sequences representing the number of instantaneous arrivals (departures) of transportation means in the
different time intervals. Such processes can equivalently be modelled either as deterministic sequences (as done, as instance, for large vessels typically characterized by a pre–scheduled timetable) or as random sequences.

Figure 1. Layout of a container terminal.

Figure 2. Queue model of the intermodal terminal operations.
When a ship reaches the terminal, containers to be unloaded wait in a queue, which has a length at time $t$ denoted by $q_1(t)$, $q_2(t)$ or $q_3(t)$, in TEUs, depending on the type of ship they are unloaded from. It can be noted again that considering a different number of ship types would only imply to insert a different number of buffers in the model.

Containers in these queues can be routed to the same transport mode (transshipment) or to the other ones. Similarly, containers arriving by road and rail waiting to be unloaded are buffered in the queues $q_4$ and $q_5$; in this case, the direction of transfer is only for ship export, as we deal with a maritime terminal.

After being unloaded from a ship, containers are located in the quay, represented by the queue $q_6$, from where some handling system moves them to the yard, in particular to the queue $q_8$, which represents the space on the ground in a lane under the yard gantry crane. As regards containers that are unloaded from trains, they are supposed to be located firstly in the ground in the railway yard, waiting in the queue $q_7$, before being loaded and handled to the yard, more precisely to $q_9$, under the yard gantry crane, where containers arriving from trucks are stocked too.

The containers temporary storage in the yard is made by means of queues $q_{10}$, $q_{11}$, $q_{12}$, $q_{13}$, $q_{14}$, $q_{15}$, $q_{16}$ and $q_{17}$. In particular, $q_{10}$, $q_{11}$ and $q_{12}$ are queues for transhipment containers, depending on the type of ship they are addressed to, $q_{13}$ and $q_{14}$ represent containers arriving by ship and going to be loaded on truck and train respectively, while $q_{15}$, $q_{16}$ and $q_{17}$ model containers that have been unloaded from truck and train and will be loaded on a ship. Analogously, queues $q_{18}$, $q_{19}$, $q_{20}$, $q_{21}$, $q_{22}$, $q_{23}$, $q_{24}$ and $q_{25}$ model the temporary wait of containers in a lane under the yard crane.

The container flow towards ships is represented by $q_{26}$, $q_{27}$ and $q_{28}$, which model the wait in the quay, while containers going to the train wait in the railway yard, in the queue $q_{29}$. Finally, $q_{30}$, $q_{31}$, $q_{32}$, $q_{33}$
and \( q_{34} \) represent the wait for the departures of the three ships, trucks and trains, when their level of storage is exactly equal to the external demand \( d_i(t) \).

Figure 3. Queue sequence for ship-to-ship container transfers.

Figure 4. Queue sequence for ship-to-truck container transfers.

In order to better clarify the meaning of each queue of the proposed model, some sketches are provided in Figs. 3 - 7, representing the queue sequences for ship-to-ship, ship-to-truck, ship-to-train, truck-to-ship and train-to-ship containers.

A complete model of transfer activities in the terminal is described by the following discrete–time equations:
Figure 5. Queue sequence for ship-to-train container transfers.

Figure 6. Queue sequence for truck-to-ship container transfers.

Figure 7. Queue sequence for train-to-ship container transfers.
\[ q_1(t + 1) = q_1(t) + \Delta T \left[ a_1(t) - u_1(t) \right] \]
\[ q_2(t + 1) = q_2(t) + \Delta T \left[ a_2(t) - u_2(t) \right] \]
\[ q_3(t + 1) = q_3(t) + \Delta T \left[ a_3(t) - u_3(t) \right] \]
\[ q_4(t + 1) = q_4(t) + \Delta T \left[ a_4(t) - u_4(t) \right] \]
\[ q_5(t + 1) = q_5(t) + \Delta T \left[ a_5(t) - u_5(t) \right] \]
\[ q_6(t + 1) = q_6(t) + \Delta T \left[ u_1(t) + u_2(t) + u_3(t) - u_6(t) \right] \]
\[ q_7(t + 1) = q_7(t) + \Delta T \left[ u_5(t) - u_7(t) \right] \]
\[ q_8(t + 1) = q_8(t) + \Delta T \left[ u_6(t) - u_8(t) \right] \]
\[ q_9(t + 1) = q_9(t) + \Delta T \left[ u_4(t) + u_7(t) - u_9(t) \right] \]
\[ q_{10}(t + 1) = q_{10}(t) + \Delta T \left[ \alpha_{8,10}(t)u_8(t) - u_{10}(t) \right] \]
\[ q_{11}(t + 1) = q_{11}(t) + \Delta T \left[ \alpha_{8,11}(t)u_8(t) - u_{11}(t) \right] \]
\[ q_{12}(t + 1) = q_{12}(t) + \Delta T \left[ \alpha_{8,12}(t)u_8(t) - u_{12}(t) \right] \]
\[ q_{13}(t + 1) = q_{13}(t) + \Delta T \left[ \alpha_{8,13}(t)u_8(t) - u_{13}(t) \right] \]
\[ q_{14}(t + 1) = q_{14}(t) + \Delta T \left[ \alpha_{8,14}(t)u_8(t) - u_{14}(t) \right] \]
\[ q_{15}(t + 1) = q_{15}(t) + \Delta T \left[ \alpha_{9,15}(t)u_9(t) - u_{15}(t) \right] \]
\[ q_{16}(t + 1) = q_{16}(t) + \Delta T \left[ \alpha_{9,16}(t)u_9(t) - u_{16}(t) \right] \]
\[ q_{17}(t + 1) = q_{17}(t) + \Delta T \left[ \alpha_{9,17}(t)u_9(t) - u_{17}(t) \right] \]
\[ q_{18}(t + 1) = q_{18}(t) + \Delta T \left[ u_{10}(t) - u_{18}(t) \right] \]
\[ q_{19}(t + 1) = q_{19}(t) + \Delta T \left[ u_{11}(t) - u_{19}(t) \right] \]
\[ q_{20}(t + 1) = q_{20}(t) + \Delta T \left[ u_{12}(t) - u_{20}(t) \right] \]
\[ q_{21}(t + 1) = q_{21}(t) + \Delta T \left[ u_{13}(t) - u_{21}(t) \right] \]
\[ q_{22}(t + 1) = q_{22}(t) + \Delta T \left[ u_{14}(t) - u_{22}(t) \right] \]
\[ q_{23}(t + 1) = q_{23}(t) + \Delta T \left[ u_{15}(t) - u_{23}(t) \right] \]
\[ q_{24}(t + 1) = q_{24}(t) + \Delta T \left[ u_{16}(t) - u_{24}(t) \right] \]
\[ q_{25}(t + 1) = q_{25}(t) + \Delta T \left[ u_{17}(t) - u_{25}(t) \right] \]
\[ q_{26}(t + 1) = q_{26}(t) + \Delta T \left[ u_{18}(t) + u_{23}(t) - u_{26}(t) \right] \]
\[ q_{27}(t + 1) = q_{27}(t) + \Delta T \left[ u_{19}(t) + u_{24}(t) - u_{27}(t) \right] \]
\[ q_{28}(t + 1) = q_{28}(t) + \Delta T \left[ u_{20}(t) + u_{25}(t) - u_{28}(t) \right] \]
\[ q_{29}(t + 1) = q_{29}(t) + \Delta T \left[ u_{22}(t) - u_{29}(t) \right] \]
\[ q_{30}(t + 1) = q_{30}(t) + \Delta T \left[ u_{26}(t) - d_1(t) \right] \]
\[ q_{31}(t + 1) = q_{31}(t) + \Delta T \left[ u_{27}(t) - d_2(t) \right] \]
\[ q_{32}(t + 1) = q_{32}(t) + \Delta T \left[ u_{28}(t) - d_3(t) \right] \]
\[ q_{33}(t + 1) = q_{33}(t) + \Delta T \left[ u_{21}(t) - d_4(t) \right] \]
\[ q_{34}(t + 1) = q_{34}(t) + \Delta T \left[ u_{29}(t) - d_5(t) \right] \]
where:

- $t = 0, 1, \ldots$ is the time step;
- $\Delta T$ is the sampling time;
- $\alpha_{i,j}(t) \geq 0$ is a sharing percentage from the queue $i$ to the queue $j$ at time interval $t$; recall that $\sum_j \alpha_{i,j}(t) = 1 \ \forall i, \forall t$;
- $a_i(t) \geq 0$ (in TEU/h) is an arrival rate of containers, $i = 1, \ldots, 5 \ \forall t$;
- $d_i(t) \geq 0$ (in TEU/h) is a departure rate of containers, $i = 1, \ldots, 5 \ \forall t$;
- $q_i(t) \geq 0$ is a state variable, representing the queue length of containers waiting to be processed (in TEU), $i = 1, \ldots, 34 \ \forall t$;
- $u_i(t) \geq 0$ is a decision (control) variable, representing the container handling rate (in TEU/h), $i = 1, \ldots, 29 \ \forall t$.

More specifically, the control variable $u_i(t)$ indicates the handling rate used to move containers from queue $q_i$ at time $t$. Quantities $u_1(t)$, $u_2(t)$, $u_3(t)$, $u_4(t)$ and $u_5(t)$ denote container unloading rates; in particular, $u_1(t)$, $u_2(t)$ and $u_3(t)$ are the unloading rates from the three different ships to the quay, $u_4(t)$ represents the unloading rate from the truck to the yard, and $u_5(t)$ is the unloading rate from the train to the ground in the railway yard. $u_6(t)$ represents the handling rate from the quay to the yard, whereas $u_7(t)$ is the handling rate from the railway yard to the yard. Moreover, $u_8(t)$ and $u_9(t)$ refer to the yard crane handling rate, from the yard lane under the crane into the storage area, and analogously $u_{10}(t), u_{11}(t), u_{12}(t), u_{13}(t), u_{14}(t), u_{15}(t), u_{16}(t)$ and $u_{17}(t)$ indicate the yard crane handling rate, from the slots where containers are stored to the yard lane where other handling systems take containers to their destinations. $u_{18}(t), u_{19}(t), u_{20}(t), u_{23}(t), u_{24}(t)$
and $u_{25}(t)$ are handling rates from the yard to the quay and $u_{26}(t)$, $u_{27}(t)$ and $u_{28}(t)$ denote container loading rates on ships. As regards containers addressed to the road, they are moved directly from the yard to a dedicated area and loaded on trucks, with a handling rate given by $u_{21}(t)$. Finally, containers going to the train are firstly moved from the yard to the railway yard ($u_{22}(t)$) and then loaded on the train by cranes that work at a rate $u_{29}(t)$.

Fig. 8 represents a simple sketch of the meaning of the handling rates in the here proposed model.

![Diagram of handling rates](image)

*Figure 8. Handling rates.*

Note that in equations (1), the coefficients $\alpha_{i,j}(t), i, j = 1, \ldots, 34, t = 0, 1, \ldots$ are considered to be a-priori known quantities. This hypothesis preserves the model linearity, which is quite useful to state significant, but still structurally simple, optimization problems. As a matter of fact, the model can be extended to consider routing coefficients as decision variables and, then, dynamic routing aspects. Anyway, being the aim of the present paper to deal mainly with strategic planning, it seems reasonable to suppose that routing is fixed.
To sum up, in the proposed model, quantities $q_i(t), i = 1, \ldots, 34$ and $u_i(t), i = 1, \ldots, 29$ represent state and control variables, respectively. The constraints on the positivity of both state and control variables are obvious but some additional requirements are necessary, as the following:

1. $u_i(t) \leq u_{i_{\text{max}}} \quad i = 1, \ldots, 29 \quad \forall t \quad (2)$
2. $q_i(t) \leq q_{i_{\text{max}}} \quad q = 1, \ldots, 34 \quad \forall t \quad (3)$
3. $\Delta T u_i(t) \leq q_i(t) \quad i = 1, \ldots, 29 \quad \forall t \quad (4)$
4. $u_1(t) + u_2(t) + u_3(t) + u_{30}(t) + u_{31}(t) + u_{32}(t) \leq QC_{\text{max}} \quad \forall t \quad (5)$

where $t = 0, 1, \ldots$. Constraints (2) enable to account that no more than the maximum handling rate is available. Constraints (3) model the obvious fact that the space in the terminal is limited. Constraints (4) impose that containers leaving the queue $i$ are not greater than those stocked in the queue itself. Finally, constraints (5), in which $QC_{\text{max}}$ represents the maximum handling rate of quay cranes, take into account that quay handling resources must be shared among different ships possibly present in the quay and among their loading and the unloading phases. Such constraints are quite important, since they allow to consider the possible resource sharing among different queues.

Note that, in the proposed model, quay cranes are the only shared resources, but it would be easy to hypothesize the sharing of any other terminal resource by simply adding constraints analogous to (5). Moreover, it is possible to relate the different terminal resources also as regards their maximum handling rates, as it will be better clarified when discussing the considered case study.
3. The optimization problem: statement in a
  receding–horizon scheme

A significant optimization problem for the system described in the previous section, will be here stated. Such a problem will be posed as an optimal control problem within a receding–horizon scheme. Then, quantities previously indicated as decision variables will be from now on equivalently denoted as decision or control variables.

The model dynamics can be expressed as

$$\mathbf{x}(t+1) = \mathbf{x}(t) + B\mathbf{u}(t) + E\mathbf{d}(t), \quad \forall t = 0,1,\ldots$$

(6)

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^m$, and $\mathbf{d}(t) \in \mathbb{R}^p$ are the state vector, control vector, and disturbance vector, respectively. Note that (1) can be more generally represented by means of (6), where $\mathbf{x}(t) \triangleq \text{col}(q_i(t), i = 1,\ldots,34)$ is the set of state variables, $\mathbf{u}(t) \triangleq \text{col}(u_i(t), i = 1,\ldots,29)$ is the control vector, and $\mathbf{d}(t) \triangleq \text{col}(a_i(t), i = 1,\ldots,5, d_i(t), i = 1,\ldots,5)$ is the vector that includes all the uncontrollable inputs, which can be regarded as disturbances. As it is clear, in the present case, $n = 34$, $m = 29$, $p = 10$.

The following constraints are assigned to the model:

$$\mathbf{x}(t) \in X$$

(7)

$$\mathbf{u}(t) \in U$$

(8)

$$\mathbf{d}(t) \in D$$

(9)

where $X \subset \mathbb{R}^n$, $U \subset \mathbb{R}^m$, and $D \subset \mathbb{R}^p$ are compact sets. The constraint sets (7)-(9) gather positivity constraints and constraints (2)-(5). As to matrices $B$ and $E$, they can be easily derived from model equations.

The optimization procedure objective of the present paper is stated as a discrete-time receding-horizon (RH) control problem. A control
scheme as the one pictorially shown in Fig. 9, is then proposed. As it is shown in a wide literature concerning RH regulators (see, as instance, (Keerthi and Gilbert, 1988; Mayne and Michalska, 1990; Michalska and Mayne, 1993; Parisini and Zoppoli, 1995; De Nicolao et al., 1998)) receding–horizon control schemes have proven to be quite effective when dealing with both linear and nonlinear systems.

A generic RH control scheme can be described as follows. When the controlled system is in the state $x(t)$ at stage $t$, an FH $N$-stage optimal control problem is solved, thus the sequence of optimal control vectors $u(t)^{FH_o}, \ldots, u(t+N-1)^{FH_o}$ is derived. The first control of this sequence becomes the control action $u(t)^{RH_o}$ generated by the RH regulator at stage $t$, that is, $u(t)^{RH_o} \triangleq u(t)^{FH_o}$. Then, a control law is obtained, as the control vector $u(t)^{FH_o}$ depends on the current state $x(t)$.

In the present case, consider the finite-horizon (FH) cost function as

$$J^{FH}(x(t), u(t, t + N - 1)) = \sum_{k=t+1}^{t+N} c^T x(k), \quad t = 0, 1, \ldots$$

(10)

where $u(t, \tau) \triangleq \text{col}(u(t), \ldots, u(\tau)), c \in \mathbb{R}^n$, $c_i > 0$, $i = 1, 2, \ldots, n$, and $N$ is a positive integer corresponding to the length of the control horizon.

Figure 9. Receding-horizon scheme.
Cost function (10) is very simple with respect to other performance indices (see, as instance (Kim, 1997; Kim and Kim, 1999)). In particular, here the objective is the minimization of a weighted sum of the queue lengths and, then, of the total transfer delay in the terminal. Note that, by suitably tuning the weights associated to the different queue lengths in the overall cost (that is, by defining coefficients \(c_i, i = 1, \ldots, n\)), it is possible to provide the defined cost function with different practical meanings. As instance, it can be assumed that the main interest of container terminal operators stands in the minimization of the time spent by ships in the terminal (as it usually happens in real case studies and as it is done in the example reported in the present paper). This can be effectively implemented in the proposed approach by giving large weights to the queues representing ship loading and unloading operations. In other cases, a greater influence could be awarded to the presence of containers in a specific area inside the terminal (by giving large values to the weighting coefficients of the corresponding queues) to take into consideration hard space limitations or high stocking costs in that area.

Then we can state the following problem.

**Problem 1.** At a generic time instant \(t\) and with reference to the state \(\overline{x}(t)\), find the FH optimal control sequence \(\{u(t)_{FH}^1, \ldots, u(t + N - 1)_{FH}^1, t = 0, 1, \ldots\}\) that minimizes cost (10) subject to (6), (7), (8), and (9).

Problem 1 has the structure of a linear mathematical programming problem that can be optimally solved by the Simplex algorithm (and, thus, by standard mathematical programming software tools). Then, thanks to the RH mechanism, once the solution of Problem 1 has been achieved, only the first optimal FH control function (i.e., the one
corresponding to \( j = t \) is actually applied to the system. This means that:

\[
\underline{u}(t)^{RH^o} \triangleq \underline{u}(t)^{FH^o}, \quad \forall t = 0, 1, \ldots \tag{11}
\]

and, more clearly, the RH control mechanism corresponds to the solution of the following problem:

**Problem 2.** At every time instant \( t = 0, 1, \ldots \), find the RH optimal control law \( \underline{u}(t)^{RH^o} \) as the first vector of the control sequence \( \underline{u}(t)^{FH^o}, \ldots, \underline{u}(t + N - 1)^{FH^o} \), solution of Problem 1 for the state \( \underline{x}(t) \).

\[\Box\]

The statement of Problem 2 does not impose any particular way of computing the control vector \( \underline{u}(t)^{RH^o} \) as a function of \( \underline{x}(t) \). Generally, there are two possibilities:

1) **On–line computation.** When the state \( \underline{x}(t) \) is reached at time \( t \), cost (10) must be minimized. Problem 1 is a linear programming problem that can be solved on–line, by considering vectors \( \underline{u}(t), \ldots, \underline{u}(t + N - 1), \underline{x}(t + 1), \ldots, \underline{x}(t + N) \) as independent variables. The main advantage of this approach is that many well-established linear programming techniques are available to solve Problem 1. On the other hand, the approach involves a high computational load for the regulator. If the dynamics of the controlled system is not sufficiently slow, as compared with the speed of the regulator’s computing system, a practical application of the RH control mechanism turns out to be unfeasible.

2) **Off–line computation.** By following this approach, the regulator must be able to generate *instantaneously* \( \underline{u}(t)^{RH^o} \) for any state \( \underline{x}(t) \) that may be reached at stage \( t \). In practice, this implies that the control law has to be a–priori (i.e., off–line) computed and stored in the regulator’s memory. Clearly, an off–line computation has advantages and disadvantages that are opposite to the ones
of an on-line computation. No on-line computational effort is requested by the regulator, but an excessive amount of computer memory may be required to store the closed-loop control law.

Obviously, on-line computation cannot be applied to the present case, since we consider a strategic planning problem.

4. Application of the model to a case study

To evaluate the effectiveness of the proposed control scheme, a simulative tool has been realized by interfacing Matlab 6.5.1 framework with Lindo 6.1 mathematical programming software. More specifically, the receding-horizon mechanism works in Matlab, while, at every time instant, the current instance of the optimization problem (Problem 1) is solved by Lindo optimization kernel. The interface between these two software frameworks is developed by means of the high level interface tools Lindo API 2.0.

The container terminal considered as a case study is a Mediterranean port in the Northern part of Italy for which a wide extension has been planned. In the present strategic planning phase, it is, then, necessary to define the main terminal characteristics by fulfilling some terminal performances defined by Authorities and reported in the following. First of all, it is supposed to have a throughput of 2000 TEU per day, that corresponds to 600000 TEU a year, assuming that the terminal works 300 days yearly (that is every day, except on Sundays and on Holidays). Traffic flows are splitted up as follows (Table I):

- from ship to ship (transhipment flows): 200 TEU a day (10%);
- from ship to truck: 500 TEU a day (25%);
- from ship to train: 400 TEU a day (20%);
Table I. Traffic flows (in TEU per day)

<table>
<thead>
<tr>
<th></th>
<th>To ship</th>
<th>To truck</th>
<th>To train</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>From ship</td>
<td>200</td>
<td>500</td>
<td>400</td>
<td>1100</td>
</tr>
<tr>
<td>From truck</td>
<td>500</td>
<td>-</td>
<td>-</td>
<td>500</td>
</tr>
<tr>
<td>From train</td>
<td>400</td>
<td>-</td>
<td>-</td>
<td>400</td>
</tr>
<tr>
<td>Total</td>
<td>1100</td>
<td>500</td>
<td>400</td>
<td>2000</td>
</tr>
</tbody>
</table>

- from truck to ship: 500 TEU a day (25%);
- from train to ship: 400 TEU a day (20%).

As described in Section II, in the proposed model, arrival and departure flows of containers are considered as exogenous random variables, that, for the considered case study, need to be defined.

In this case study, three typologies of ships are considered, so that ship arrival rates are represented by three different quantities, $a_1(t)$, $a_2(t)$ and $a_3(t)$:

- $a_1(t)$ denotes the arrival rate of deep-sea vessels from which 4800 TEU are unloaded; moreover, it is supposed that only one of these vessels arrives every week and this arrival is considered deterministic, so that each arrival of a deep-sea vessel is equal to the previous one, but it happens exactly one week after;

- $a_2(t)$ denotes the arrival rate of feeder vessels from which 1000 TEU are unloaded; only one of these ships is assumed to arrive every week and this arrival is a random variable, because it is possible that this vessel arrives at every hour in the week;

- $a_3(t)$ represents the arrival rate of other feeder vessels from which 400 TEU are unloaded; for these kinds of ships, two arrivals are supposed to happen every week, in a random way, because one of
these vessels can arrive in the first half of the week and the other in the second half.

To be noted that the sum of $a_1(t)$, $a_2(t)$ and $a_3(t)$ corresponds to a container flow from ship equal to 6600 TEU per week (in accordance with data presented in Table I, indicating 1100 TEU per day unloaded from ship). A possible configuration of $a_1(t)$, $a_2(t)$ and $a_3(t)$ is shown in Fig. 10, respectively with a dashed-dotted, dashed and continuous line.

![Figure 10. Ship arrivals.](image)

The truck arrival rate is equal to 500 TEU a day, split in 31 TEU every hour, on average, for 16 hours a day, because the gate is supposed to be closed during the night. The truck arrival rate $a_4(t)$ is therefore assumed as an integer random variable, that could vary between 26 and 36 TEU/h, while it is posed equal to zero for 8 hours a day. A possible configuration of $a_4$ is shown in Fig. 11.

The train arrival rate is equal to 400 TEU a day; since each train is supposed to carry 40 TEU (corresponding to 20 flat wagons carrying 2 TEU each), this means that 10 trains arrive every day at the terminal, on average, again in 16 hours a day. The value of $a_5$ is an integer random
number between 35 and 45 TEU/h and the number of train arrivals in the day time period is random too, with an average of 10 arrivals a day. A possible configuration of $a_5$ is shown in Fig. 12.

As regards the departure rates by these transport modes, that is $d_1(t)$, $d_2(t)$, $d_3(t)$, $d_4(t)$ and $d_5(t)$, their patterns are analogous to the corresponding arrival rates.
Moreover, as regards the routing coefficients, their evaluation has been based on the previously described split of traffic flows; it can be noted that, working at a strategic planning level, it is reasonable to consider such coefficients as time–invariant quantities, that is \( \alpha_{i,j}(t) = \alpha_{i,j}, \forall t \), as follows:

\[ - \alpha_{8,10} = 0.13; \quad \alpha_{8,11} = 0.03; \quad \alpha_{8,12} = 0.02; \quad \alpha_{8,13} = 0.46; \quad \alpha_{8,14} = 0.36; \]
\[ - \alpha_{9,15} = 0.73; \quad \alpha_{9,16} = 0.15; \quad \alpha_{9,17} = 0.12. \]

The maximum handling rates \( u_{i\text{max}} \) (in TEU/h) can be related to one another, to represent the fact that some handling resources have the same dimensions or belong to the same flow (even if they are not shared). In particular, in the present model, all the maximum handling rates depend on the following quantities:

\[ - u_{1\text{max}} = 150; \]
\[ - u_{2\text{max}} = 105; \]
\[ - u_{3\text{max}} = 45; \]
\[ - QC_{\text{max}} = 150; \]
\[ - u_{4\text{max}} = 40; \]
\[ - u_{5\text{max}} = 50. \]

\( QC_{\text{max}} \) has been fixed equal to 150 TEU/h, because 5 cranes are supposed to be used in the quay, splitted in 3 double-trolley cranes, each working at a 35 TEU/h rate, and 2 single-trolley cranes, able to handle 22.5 TEU/h each. Moreover, the deep-sea vessels can be loaded and unloaded by all the 5 cranes (thus, \( u_{1\text{max}} = 150 \)), while the two feeder vessels can be operated, respectively, by the 3 double-trolley cranes (\( u_{2\text{max}} = 105 \)) and the 2 single-trolley cranes (\( u_{3\text{max}} = 45 \)).
Table II. Maximum handling rates $u_{i\text{max}}$.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$QC_{\text{max}}$</th>
<th>$u_{1\text{max}}$</th>
<th>$u_{2\text{max}}$</th>
<th>$u_{3\text{max}}$</th>
<th>$u_{4\text{max}}$</th>
<th>$u_{5\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 8</td>
<td>$QC_{\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7, 14, 22, 29</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$u_{5\text{max}}$</td>
</tr>
<tr>
<td>10, 18</td>
<td>-</td>
<td>0.18$u_{1\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11, 19</td>
<td>-</td>
<td>-</td>
<td>0.18$u_{2\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12, 20</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.18$u_{3\text{max}}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13, 21</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$u_{4\text{max}}$</td>
</tr>
<tr>
<td>15, 23</td>
<td>-</td>
<td>0.82$u_{1\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16, 24</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.82$u_{2\text{max}}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17, 25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.82$u_{3\text{max}}$</td>
<td>-</td>
</tr>
<tr>
<td>26</td>
<td>-</td>
<td>$u_{1\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>27</td>
<td>-</td>
<td>-</td>
<td>$u_{2\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>28</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$u_{3\text{max}}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

As just explained, all the other maximum handling rates $u_{i\text{max}}$ depend on the previously defined parameters, as shown in Table II, where each row represents the maximum handling rate associated to queue $i$ and indicates the dependence on the parameter in the corresponding column. The only exception is for $u_{9\text{max}}$ which can be obtained as the sum of two quantities, as follows:

$$u_{9\text{max}} = u_{4\text{max}} + u_{5\text{max}}$$

As regards the maximum queue lengths $q_{i\text{max}}$, they have been posed equal to very large numbers, since in this planning phase, the various parts of the terminal must still be sized. What is expected is that the queue lengths found with the optimization here carried on provide just the indication of the optimal maximum sizes for the different physical areas represented by the different queues.
As regards the cost coefficients $c_i$ in the objective function, they have been fixed as follows:

- $c_1 = c_{30} = 2.2$;
- $c_2 = c_{31} = 2.1$;
- $c_i = 2.0$, $i = 3, 4, 5$, $c_i = 2.0$, $i = 32, 33, 34$;
- $c_i = 1.8$, $i = 6, \ldots, 9$, $c_i = 1.8$, $i = 18, \ldots, 29$;
- $c_i = 1.0$, $i = 10, \ldots, 17$.

In this way, the most weighted queues are those referred to loading and unloading operations, so that when a carrier arrives, it is unloaded or loaded as soon as possible. Moreover, among ship loading and unloading operations, the weight is larger for deep-sea vessels ($c_i = 2.2$), and lower for feeder vessels ($c_i = 2.1$ and $c_i = 2.0$ respectively); in this way, quay cranes, which are shared among the three different ships, tend to ”favour” larger ships than feeder vessels. Moreover, queues representing containers in the quay, in the railway yard and in the lanes under yard cranes are weighted more ($c_i = 1.8$) than those corresponding to containers in the yard ($c_i = 1.0$), because containers must remain in the yard just until it is necessary to move them to other areas of the terminal.

A further requirement for the considered terminal is that the mean time spent by containers in the terminal is of about 6 days and this has been realized in the simulation tool by suitably defining the system initial conditions.

In the following, some simulation results will be presented, regarding an overall simulation of 1000 time intervals (hours), while the FH time horizon is equal to 48 time intervals (hours).

Fig. 13 shows containers in the queues $q_1$ and $q_{30}$, in a continuous and dashed line respectively, that represent containers waiting to be
unloaded from and loaded on the deep-sea vessel. Looking at this graph, it is quite clear that when a ship arrives, it is unloaded as soon as possible and, then, it is loaded. The different slopes of these lines show the fact that in some time intervals it is not possible to use the quay crane maximum handling rate, because it is also used to load/unload other ships. The same meaning can be associated with Fig. 14, in which the control variable $u_1$ associated with queue $q_1$ is equal to its maximum.
value (150 TEU/h) when the deep-sea vessel needs to be unloaded or loaded and there are not other ships in the quay. A further explanation of quay crane sharing can be found in Fig. 15, where all the control variables corresponding to ship loading and unloading operations are shown.

Figure 15. Control variables $u_1$ (dashed-dotted), $u_2$ (dashed), $u_3$ (continuous), $u_{26}$ (thick dashed-dotted), $u_{27}$ (thick dashed), $u_{28}$ (thick continuous).

Figure 16. Queue $q_5$. 
Figs. 16 and 17 show the state and the control variable, respectively, for the queue $q_5$, that is the queue for containers arriving by train and waiting to be unloaded. These graphs have the same patterns, which is exactly equal to that of train arrivals (Fig. 12); this is caused by the fact that, when a train arrives, carrying about 40 TEU, cranes in the railway yard are able to unload it in less than 1 hour, since the maximum handling rate $u_{5\text{max}}$ has been fixed to 50 TEU/h.

Figure 17. Control variable $u_5$.

Figure 18. Containers in the yard.
Fig. 18 shows containers stored in the yard (sum of $q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}$ and $q_{17}$), while Figs. 19-21 represent containers stored in the yard, but divided in transhipment, export and import flow, respectively.
5. Conclusions

In this paper, a model for intermodal container terminals has been proposed; this model is made by a set of queues, which represent container positions inside the terminal, while the dynamic evolution of the system is represented by means of discrete-time equations. The model is used as the basis for stating and solving an optimization problem specifically related to the strategic planning of maritime container terminals, even if other decisional phases could be equivalently taken into account by adopting the same methodology. The optimization problem is posed as an optimal control problem and solved within a receding-horizon scheme.

Present and future research is devoted to better establishing the performance of the proposed approach as regards the definition of an a–priori feasibility analysis of the optimization problems faced in the receding–horizon scheme and the analysis of the involved computational burden.
References


