Network optimization – formulation exercises

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1

At the end of 2017, a company acquired a new vehicle for the delivery of goods. At the end of each subsequent year, the company can decide to either sell the vehicle and buy a new one, or keep the current vehicle. Maintenance costs increase as time goes by. The company wants to decide the purchase/maintenance policy for the next four years (until the end of 2021) so to minimize overall costs. The price of a new vehicle at the end of each year is shown in the first table. In any case, we know that at the beginning of 2022 a new vehicle will be bought.

year	Vehicle price
2018	12000
2019	13000
2020	14000
2021	15000

The second table reports maintenance costs (euros) to be paid if the vehicle is kept for one, two, three or four years, and the price at which it is possible to sell a vehicle which is one, two, three or four years old.

	Maintenance costs	Vehicle value
one year	2000	11000
two years	3000	10000
three years	5000	8000
four years	8000	6000

The problem is to decide the purchase/maintenance policy that minimizes overall costs. Formulate and solve the problem as a network optimization problem.

2

A company wants to launch a new product on the market. This requires performing a series of activities, reported in the table. For each activity i, we report the predecessors (i.e., the activities that must be completed before i starts) and the duration in days.

Activity	Predecessors	Duration (days)
A (Workers training)	-	6
B (Raw materials purchase)	-	9
C (Raw material transportation)	A	2
D (Production component 1)	C, B	7
E (Production component 2)	C, B	7
F (Testing component 2)	E	1
G (Final assembly)	D, F	12

Represent the project by a suitable network and determine project duration solving a network optimization problem.

3

A company that builds engines for aircraft plans its production for the next three months. At the beginning of January, one engine is in inventory. The number of engines which are to be delivered at the end of January, February and March is respectively 2, 1 and 0. In addition, at the beginning of April, the company wants to have one spare engine in inventory. The demand at the end of a given month can be satisfied either by the production of the current month and/or withdrawing from inventory. At most two engines can be produced each month. The cost to produce 0, 1 or 2 engines in a given month is respectively €10,000 (due to plant maintenance costs), €18,000 and €20,000. If, at the end of a month, 1 or 2 engines are left in inventory, the holding costs reported in the table are incurred (if at the end of a month there are no engines in inventory, then holding costs are 0).

month	no. of engines	Inventory costs
January	1	4000
	2	7000
February	1	5000
	2	7500
March	1	5500
	2	8000

The problem is to decide how many engines to produce each month in order to meet the demand and minimize the overall costs. Formulate the problem in terms of a network optimization problem.

4

You have decided to organize a dinner at your house. Since you are too busy studying for the exam of Network Optimization, you asked your friends to cook for you. After much thought on the cooking skills of your friends, you have come to draw up the following table, where the number indicates your judgment on the corresponding dish prepared by your friends (the higher the better).

friend	Starter	First c.	Second c.	Side dish	Dessert
Andrea	7	6	5	7	8
Barbara	6	8	7	6	5
Chuck	6	5	4	4	8
Doriana	7	8	6	6	6
Ellis	5	6	7	5	0
Frank	7	8	8	8	6
Geneve	7	7	5	5	6

The problem is to decide *if* and *what* should each friend prepare, considering that your dinner will consist of one dish of each type (i.e., a starter, a first course, a second course, a side dish and a dessert) and you do not want to ask anyone to prepare more than one dish. Formulate the problem in terms of network optimization and solve it by using an appropriate algorithm.

5

A supercomputing center has a set of processors (1,2, ..., 10). You must perform a set of jobs (A, ..., I). All jobs require the same time (one hour) and two processors simultaneously, which during this time cannot be devoted to any other job. Formulate and solve (by means of an appropriate algorithm) the problem of executing the maximum number of jobs, supposing to have only one hour available.

jobs	processors
A	1,6
B	1,7
C	1,10
D	2,6
E	3,6
F	4,10
G	5,8
H	5,9
I	1,9

6

A set of machining centers (A, B, C) must perform operations of various types (1, 2, 3, 4) on a set of mechanical parts. Each type of operation can be performed on a subset of centers. The demand for each type of operation is expressed in minutes. Each operation type can be divided in any possible way among the machining centers enabled to perform it (e.g., x minutes of operation type 1 can be assigned to A and the remaining $50 - x$ to B).

operations	centers	time
1	A,B	50
2	A,C	20
3	A,B,C	10
4	B	70

Considering that in the next shift, centers A, B and C will be available for 30, 80 and 40 minutes respectively, the problem is to determine whether or not is it possible to complete all the processes during the next shift. Formulate the problem in terms of network optimization and solve it by means of a suitable algorithm.

7

You are the owner of a banquet hall, and you received a number of requests for a given day. Each request specifies the time window in which the room is requested, and the number of seats needed (people). The problem is to select a subset of requests such that the room is never used by two groups simultaneously and the number of people served in a day is maximized. Formulate the problem in terms of network optimization and solve it through an appropriate algorithm.

request	start time	end time	number of customers
1	8	10	60
2	9	11	50
3	10	12	20
4	10	13	50
5	11	12	40
6	11	13	60
7	12	14	50
8	12	15	70
9	13	16	20

8

It's Monday morning. In a furniture factory, a painting station must process four batches of kitchen furniture, having different colors. Since switching from one color to another takes a long time for equipment cleaning, each batch must be done on a different day. A batch i is characterized by a *weight* w_i and a *due date* d_i. If a batch is painted no later than d_i, it is delivered on time. Otherwise, for each day of delay with respect to d_i, w_i euros of penalty are paid.

i	w_i	d_i
1	80	Wednesday
2	160	Tuesday
3	200	Tuesday
4	120	Tuesday

The problem consists in deciding on which day should each batch be painted, in order to minimize penalty costs. Formulate the problem in terms of network optimization and solve it by a suitable algorithm.

9

The coach of the national swimming team must decide the composition of the relay for the 200 meters medley at the next olympic games. The coach has 6 athletes. The table indicates the personal record (on 50 meters, in seconds) of each swimmer for each of the four styles.

swimmer	backstroke	breaststroke	butterfly	free-style
1	19	22	16	13
2	17	25	17	14
3	20	21	15	12
4	20	22	18	13
5	18	20	18	14
6	18	21	19	15

The problem is to choose the most competitive team, assuming that each athlete is able to repeat his personal record. Formulate the problem in terms of network optimization and solve it by a suitable algorithm.

10

The authority for telecommunications decides to sell frequencies of a new band for the video signal. The bandwidth ranges from 903 MHz to 951 MHz, with an interval of 6 MHz between one frequency and the next. To participate in the call, one must specify an *interval* of frequencies (inclusive), and make a bid for this interval. At the expiry of the notice, the following bids are received:

operator	from	to	bid (millions of euros)
A	903	909	13
B	903	915	15
C	909	915	12
D	915	921	10
E	915	933	32
F	921	927	8
G	921	939	40
H	927	939	25
I	927	945	31
J	933	945	22
K	939	951	29
L	945	951	15

Each frequency can be assigned to at most one operator. Formulate in terms of network optimization the problem of determining which bids to accept to maximize revenues.

11

Some cameras need to be deployed to monitor the corridors of a museum. The following pictures show two floors of the museum, in which arcs correspond to corridors and nodes to meeting points of different corridors. A camera placed in a node is able to monitor all

the corridors corresponding to the edges that are incident in the node. The problem, at each floor, is to determine the minimum number of cameras (and their locations) in order to monitor all the corridors.

1) Formulate the problem as an ILP

2) Is it possible to solve it as an LP, in the two cases?

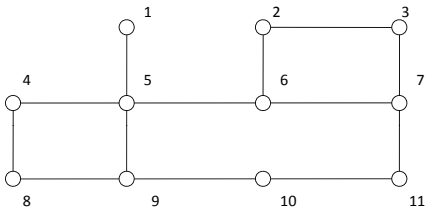

Figure 1: Floor 1 in the museum of Problem 11.

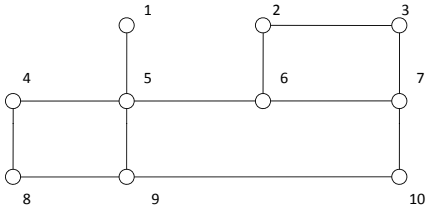

Figure 2: Floor 2 in the museum of Problem 11.

12

The roads to go from Ice Creek to Snowville have been invested by a violent blizzard, and this may have blocked them. Considering that the local road network can be represented by an oriented graph $G(N, A)$, the police announced the probability p_{ij} for an edge (i, j) to be open. (Assume that the probabilities are all independent.) The problem is to find the path from Ice Creek to Snowville that maximizes the probability of actually reaching Snowville. Formulate the problem as a shortest path problem.

13

A group of 100 tourists must be transferred from Rome to Sydney. Unfortunately, given the high season and the close date of departure, they cannot be transferred by a single direct flight. After a careful research, you have gathered some information on available flights and the related number of seats still available. Assuming that your goal is to get all the tourists to Sydney, formulate the problem in terms of an appropriate problem of network optimization on graphs, and apply an appropriate algorithm to determine whether this possibility exists.

leg	available seats
Roma-Cairo	60
Roma-Katmandu	50
Cairo-Katmandu	40
Cairo-Tokyo	50
Katmandu-Delhi	20
Katmandu-Seul	30
Delhi-Tokyo	15
Tokyo-Seul	20
Seul-Sydney	60
Tokyo-Sydney	40

14

A retailer of used stereo components has 6 speakers (A, B, C, D, E, F). The frequency response of each speaker has been measured for a set of eight reference frequencies (1, ..., 8). The table shows (with an 'X') the interval of frequencies the response of each speaker is acceptable (*nominal interval*). The manufacturer wants to sell these speakers in pairs, but two speakers can be sold as a pair only if the nominal intervals of the two speakers have at least 5 reference frequencies in common.

diffusore	1	2	3	4	5	6	7	8
A		X	X	X	X	X		
B			X	X	X	X	X	X
C	X	X	X	X	X			
D	X	X	X	X	X	X	X	
E			X	X	X	X	X	
F	X	X	X	X	X	X		

1) The problem is how to form the maximum number of speaker pairs. Formulate the problem as a network optimization problem.

2) Can the problem be solved via LP?

15

A new commercial center is to be built on a very bumpy area, in which there are many valleys and hills. Dividing the area into ten zones, it can be specified for each zone the current *height* with respect to the final street level. These heights are measured by the number of truckloads of earth over or below street level (so, for instance, two truckloads must be added to zone 1 to reach street level, while eight truckload must be removed from zone 6).

zone	1	2	3	4	5	6	7	8	9	10
height	-2	5	0	-1	4	-8	3	-2	-1	2

The cost for moving a truckload of earth from zone i to zone j is proportional to the distance between the two zones, indicated in the following matrix. (Note that the distances satisfy the triangle inequality, i.e., $c_{ij} \leq c_{ik} + c_{kj}$ for any i, j, k.)

$$\begin{pmatrix} 0 & 1.4 & 3.2 & 5.1 & 3 & 1.4 & 5 & 5 & 2.8 & 5 \\ 1.4 & 0 & 2.8 & 4 & 2.2 & 2 & 3.6 & 3.6 & 1.4 & 4.1 \\ 3.2 & 2.8 & 0 & 2.8 & 1 & 2 & 4.1 & 5 & 3.2 & 2.2 \\ 5.1 & 4 & 2.8 & 0 & 2.2 & 4.5 & 2.2 & 3.6 & 3.2 & 1 \\ 3 & 2.2 & 1 & 2.2 & 0 & 2.2 & 3.2 & 4 & 2.2 & 2 \\ 1.4 & 2 & 2 & 4.5 & 2.2 & 0 & 5 & 5.4 & 3.2 & 4.1 \\ 5 & 3.6 & 4.1 & 2.2 & 3.2 & 5 & 0 & 1.4 & 2.2 & 3.2 \\ 5 & 3.6 & 5 & 3.6 & 4 & 5.4 & 1.4 & 0 & 2.2 & 4.5 \\ 2.8 & 1.4 & 3.2 & 3.2 & 2.2 & 3.2 & 2.2 & 2.2 & 0 & 3.6 \\ 5 & 4.1 & 2.2 & 1 & 2 & 4.1 & 3.2 & 4.5 & 3.6 & 0 \end{pmatrix}$$

The problem is to move the earth so as to pave the land at minimal cost.

1) Formulate the problem as a network optimization problem.

2) Can the problem be solved via LP?

16

Six nuclear power stations (A, B, C, D, E, F) produce waste, which must be properly disposed of in a disposal center. There are four disposal centers, each able to treat only certain types of waste produced. In particular, the table indicates the quantity (per year) of waste produced (thousands of tonnes) and the centers that are able to treat them. Each center can dispose of up to 30 tonnes of waste per year. Given such capacity constraint, the problem is to determine whether or not is it possible to dispose of the waste produced by the six power stations. Formulate the problem as an appropriate network optimization problem and solve it by a suitable algorithm.

	amount of waste produced	enabled centers
A	25	1,3
B	10	2,4
C	20	2,3
D	30	1,2
E	20	1,4
F	15	3,4

17

A project consists of 6 phases (A, B, C, D, E, F), each having known duration. There are precedence constraints among phases, in the sense that some may only start after the end of others. These data are summarized in the table.

phase	duration	must precede...
A	20	C, D, E
B	15	D, E
C	25	F
D	30	F
E	40	–
F	15	–

Draw the project network and compute its duration by means of a suitable algorithm.

18

An aggregator of electricity must collect a certain quantity of energy at certain times of next day (between 12 and 16). To do this, it can rely on some users (A, ..., E) that are willing to reduce their consumption to a certain value of total kWh in a certain time interval, as indicated in the table. (Assume that the time unit is 1 hour.) For example,

user A is willing to reduce her consumption by up to 2 kWh, as long as such reduction takes place between 12 and 15. To do this, users request a money compensation from the aggregator (prize).

user	kWh	interval	prize (euro/kWh)
A	2	12–15	0,50
B	1	13–15	1
C	2	14–16	0,75
D	3	15–16	1
E	2	12–14	0,80

The aggregator must satisfy the demand specified in this table.

hour	requested kWh
12–13	1
13–14	1
14–15	2
15–16	3

The problem is to satisfy all the electricity demand in the various hours minimizing the costs for the aggregator. Formulate the problem as network optimization.

19

A set of n currencies is given. For each pair (i, j) of currencies, a value p_{ij} is given, which expresses the quantity of currency j one can buy with one unit of currency i. (For example, if i= euro and j = dollar, $p_{ij} = 1.4$ indicates that with one euro it is possible to buy 1.4 dollars.) We assume that for each i, j, $p_{ij} = 1/p_{ji}$. Assuming that there are no transaction fees, a speculator wants to find out whether is it possible to identify a cycle of currencies $i_1 \to i_2 \to i_3 \to \ldots \to i_k \to i_1$ allowing one to achieve a net gain simply buying $i_2$with currency i_1, i_3 with currency i_2, ..., and finally buying back i_1 using currency i_k. Explain how to formulate the problem in terms of network optimization.

20

A video system must estimate the speed of four identical objects moving on a plane. At a certain instant, a picture of the objects is taken. The objects have the positions indicated in the figure as 1, 2, 3 and 4. After a few seconds, another picture is taken, and the same objects appear in the positions indicated by a, b, c, d. The problem is to identify the position of the four original objects, in the second photo. It is believed that the

most plausible identification is the one that minimizes the sum of the Euclidean distances between the new and the old position of each point.

Formulate the problem (without solving it) in terms of network optimization.

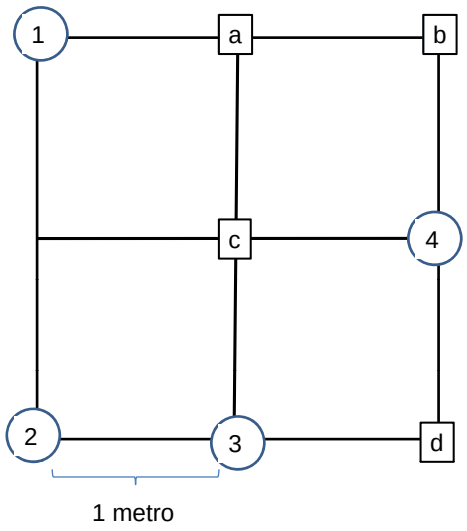

Figure 3: Objects on the plane for Problem 20.

21

In a production plant there are 3 machines, which must be used to produce 7 steel dies (A, ..., G). The molds are all identical, but come from different customer orders, and have therefore different importance (weight). Furthermore, the three machines are not identical: while machine 1 takes 1 hour to produce a mold, machine 2 takes 40 minutes and machine 3 (the fastest) takes 30 minutes. For each mold, a *due time* (expressed in hours from now) is specified, within which the mold should be completed. For each mold whose completion time exceeds the due time, a penalty occurs equal to the delay (in minutes) multiplied by the weight. The problem is to allocate the molds to the machines and sequence them so as to minimize the overall penalty. Formulate (without solving it) the problem as an appropriate network optimization problem.

die	weight	due time
A	2	1 hour
B	3	1 hour and a half
C	2	2 hours
D	5	1 hour and a half
E	3	2 hours
F	4	1 hour
G	2	1 hour and a half

22

Grunt, the caveman, possesses a club and a number of shells. Grunt can barter its objects as shown below:

Club + 3 shells = Dog
Club + 10 shells = Ox
Dog + 6 shells = Ox
Club + 17 shells = Canoe
Dog + 13 shells = Canoe
Ox + 6 shells = Canoe
Club + 50 shells = Palafitte
Dog + 45 shells = Palafitte
Ox + 35 shells = Palafitte
Canoe + 15 shells = Palafitte

What is the minimum number of shells Grunt should pay (and which barters should perform) to get a palafitte? Formulate and solve the problem as a network optimization problem.

23

A set of cans, each containing some chemicals and having a certain weight, must be shipped by a truck. The chemicals A and B, as well as the chemicals C and D, cannot be put on the same truck because, in case of an accident, they might come into contact and explode. Considering that a truck cannot carry more than 500 kg, formulate (without solving it) in terms of network optimization the problem of determining the minimum number of trucks needed to transport all the cans.

can	contains	weight
1	A, C, E	270
2	C, E	260
3	A, D, E	190
4	B, C	260
5	B, E	250
6	A, E	220
7	B, E	260
8	D	250

24

A company produces boilers, and must plan its production for the first five months of the year. For each month the following quantities are specified: a unit production cost p_i, a demand d_i (i.e., the number of boilers which are to be delivered at the end of month i), and a production capacity K_i, i.e., the maximum number of boilers which can be produced during month i. The boilers produced and not sold at the end of a month are stored (and therefore may be used to meet the demand of the coming months). For each boiler remaining in inventory at the end of month i there is an inventory cost h_i. The company does not want to have boilers in inventory at the end of May.

i	d_i	p_i	K_i	h_i
January	20	300	30	15
February	10	250	30	10
March	15	320	30	8
Aprile	20	320	35	12
Maggio	10	350	35	–

The problem is to plan production in order to minimize overall costs, while respecting capacity constraints and satisfying all the demand. Formulate (without solving it) the problem in terms of network optimization.

25

A steel producer makes use of various metal bars, all having the the same section but different length (class). The price of a bar of class i is equal to c_i, while creating a warehouse to accommodate bars of class i implies a fixed cost K_i (regardless of how many bars it will host; suppose for simplicity that there are no capacity limits). Once created, the warehouse can only accommodate bars of the corresponding class. However, a bar of

a certain class i can be *always* used in place of a bar of lower length, i.e., of class $k < i$. The number d_i of bars of class i that will be needed by production in the next term is known. The problem is to decide for which classes of bars to create a warehouse, in order to minimize overall costs. Formulate the problem in terms of network optimization and solve it by means of an appropriate algorithm.

classe	c_i	d_i	K_i
1	15	8	100
2	20	6	170
3	30	5	220
4	35	9	250
5	40	10	300

26

Every year Snowville hosts the traditional fair of stewed ice cream. To entertain the visitors, in the morning, at each of six spots of the city (shown in the figure as A, B, C, D, E, F), there is a group of street artists. At the end of the morning, the organizers want to move each of the six groups to a different spot, in such a way that the total distance covered by the artists is minimized (in the map, lines represent the roads). Formulate the problem of determining how the artists should be moved as a network optimization problem, and solve it by means of an appropriate algorithm.

27

A car-pooling center receives a number of requests (*trips*), between 8am and 5pm. Each trip starts and ends in the center, and is defined by a start time and a finish time. You have a car, that you wish to rent to perform some trips. Since the centers pays you proportionally to the time spent traveling between 8am and 5pm, you wish to select the subset of trips that maximizes the time spent traveling. Consider that after each trip, due to some paperwork the car is not available for a new trip for 30 minutes.

Formulate the problem as a network optimization problem, and solve it by means of an appropriate algorithm.

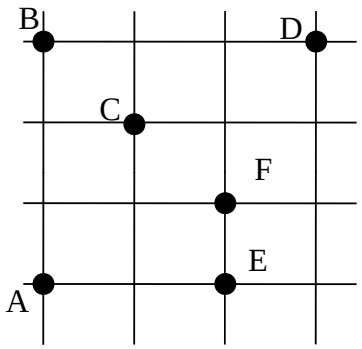

Figure 4: City map for Problem 26.

trip	start time	finish time
1	8	10
2	9	11
3	8,30	10,30
4	10,30	13
5	11	12,30
6	11	14
7	11,30	13,30
8	12	14
9	14	15,30
10	15,30	16,30
11	16	17

28

A cargo company owns a tramp steamer, who can travel among a set N of four ports, transporting goods and passengers. For each pair of ports (i, j), a certain travel *time* t_{ij} (hours) is known, as well as the *profit* p_{ij} (thousands of euros) if the trip is done. These data are shown in matrices T and P:

$$T = \begin{pmatrix} - & 2 & 3 & 3 \\ 6 & - & 3 & 3 \\ 6 & 4 & - & 3 \\ 4 & 5 & 4 & - \end{pmatrix} \quad P = \begin{pmatrix} - & 5 & 4 & 2 \\ 7 & - & 5 & 2 \\ 5 & 5 & - & 4 \\ 3 & 3 & 2 & - \end{pmatrix}$$

A subtour is a cycle involving some cities (possibly, all) of N. Given a subtour W, the *profit per time unit* is defined as

$$\frac{\sum_{(i,j)\in W} p_{ij}}{\sum_{(i,j)\in W} t_{ij}}.$$

The managers of the company wish to know whether or not is it possible to define a subtour having a profit per time unit larger than 1000 euros/hour. Formulate the problem in terms of network optimization and solve it by means of a suitable algorithm.

Hint: observe that the problem is equivalent to finding a subtour W such that

$$\sum_{(i,j)\in W} t_{ij} < \sum_{(i,j)\in W} p_{ij}.$$

29

The inventory of a hospital has to be reorganized. Each drug type has to be allocated exactly one *container*, i.e., each container will be devoted to a single drug (we assume that the capacity of any container is large enough to host any number of packs of a given drug). However, the same container type can be chosen for different drugs. A new set of standard containers has to be adopted, different from each other on the basis of their *width*. A container of type j, having width w_j, can contain any drug i whose packs have width $\ell_i \leq w_j$. There are 6 container types, characterized by the following data: width w_j, per-unit cost c_j, fixed cost f_j. The latter is independent of the number of containers ordered, and corresponds to shipping costs (the containers are produced by different firms). So if, say, containers of type D are chosen for two drugs, the corresponding contribution to overall costs is $f_D + 2c_D = 140 + 2 * 25 = 190$.

Complete data are reported in the tables. Notice that as the width grows, also per-unit and fixed costs grow.

The problem is to decide which container type should be used for each drug, so that overall costs are minimized. Formulate the problem in terms of network optimization and solve it by means of a suitable algorithm.

drug	box width ℓ_i (cm)
1	10
2	12
3	14
4	16
5	18
6	20
7	22
8	25
9	27
10	30

container type	w_j (cm)	f_j (€)	c_j (€/container)
A	10	100	10
B	15	120	15
C	17	130	20
D	20	140	25
E	26	150	28
F	32	160	30

30

Consider the acyclic graph in the figure, in which a topological numbering is specified. The problem is to find the shortest path from 1 to 10 *consisting of exactly 5 arcs*. Show that this problem can be solved as an ordinary shortest path on another appropriately defined acyclic graph.

31

The graph represents a road network connecting s and t. The policemen learnt that a truck will leave s to deliver illegal drugs in t. A patrol located on a freeway (i.e., an arc) is able to check all the trucks passing by. The problem is to determine the minimum number of patrols needed to intercept the suspect truck. Formulate the problem as graph optimization and solve it by a suitable algorithm.

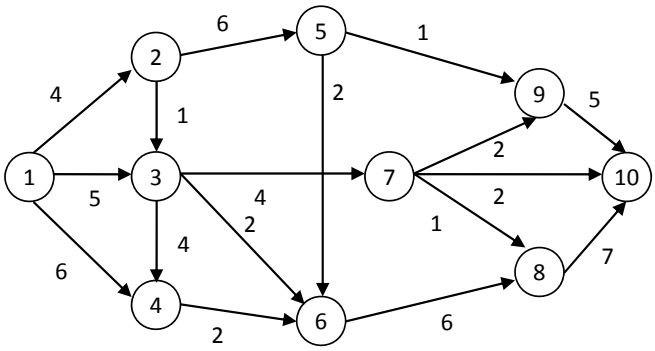

Figure 5: Graph for Problem 30.

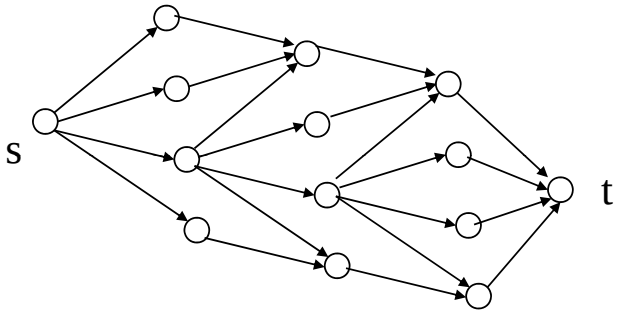

Figure 6: Graph for Problem 31.

32

A flexible cell produces motherboards. In the cell, there are two identical working spaces, in each of which a robot performs pick-and-place assembly operations. The robots pick each component from the corresponding *feeder*. There is exactly one feeder for each component type. There are 7 different component types (and hence feeders). The table lists the components required by each batch (e.g. producing batch 1 requires accessing feeders A, B and C). Since each feeder can be accessed by only one robot at a time, the two robots can work in parallel on two different batches of motherboards if and only if these have no components in common. Each batch requires one hour to be assembled. Batches cannot be preempted.

The problem is to assign batches to the two robots and sequence them so that the

overall makespan is minimized. Formulate the problem as graph optimization.

batch	feeders
1	A, B, C
2	A, B, E
3	A, C, D
4	A, E, F
5	A, D
6	C, D, G
7	B, F, G
8	D, E, G
9	C, E, G
10	B, E, G

33

The picture shows the map of a city quarter, consisting of four (horizontal) avenues and four (vertical) streets. Local police wants to locate 4 patrols of agents to control all the roads. A patrol located in an intersection controls both the street and the avenue intersecting in the point. For each intersection, a *score* is given, expressing the hazard for a patrol to be attacked by gangs of thugs (1=safe neighborhood, 10=extremely dangerous neighborhood). The problem consists in deciding where to locate the 4 patrols, minimizing the total score. Formulate the problem and solve it through an appropriate algorithm.

34

An airline company decides to organize flights connecting certain city pairs, namely: A–F; A–G; A–H; B–G; B–I; C–F; C–G; C–J; D–H; D–J; E–H; E–I.

The problem is to decide in which cities to open an office so that, for each flight, there is one office in at least one of two cities.

Formulate the problem of minimizing the number of offices as a network optimization problem.

35

A company performs the customization of motorhomes according to customers specifications. It has two plants, A and B. In A, highly specialized personnel can customize a motorhome in 2 days. In B there are less resources, and the customization of a motorhome requires 3 days. At time 0, the orders indicated in the table are pending. For each order,

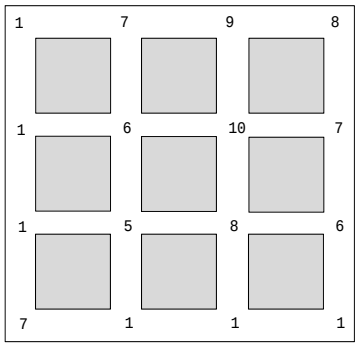

Figure 7: City quarter with safety scores.

three quantities are specified: Ideal due date, daily penalty for late delivery (w_i^+ euros for each day of tardiness), daily penalty for early delivery (w_i^- euros euros for each day of earliness). The problem is to allocate the orders to the two plants so that total costs are minimized. Formulate the problem as a network optimization problem and solve it through an appropriate algorithm.

order	due date	w_i^+	w_i^-
A	4	100	10
B	4	250	20
C	5	200	25
D	4	150	30
E	5	450	10
F	5	300	20
G	6	50	30

36

On the 25 squares of a 5x5 chessboard one wants to locate 5 rooks (of the game of chess), so that no rook can threaten another. In other words, rooks must be located so that no two rooks occupy the same row or column of the chessboard. Each square has a *score*. The problem is to determine a feasible location of the 5 rooks minimizing the score of the occupied squares. Formulate the problem as a network optimization problem and solve it through an appropriate algorithm.

3	6	10	6	3
6	5	6	5	6
10	6	12	6	10
6	5	6	5	6
3	6	10	6	3

37

The city of Petaluma wants to outsource the schoolbus service. Three companies responded to a public call, bidding for certain routes. The table shows the set of received bids. No more than two lines can be outsourced to the same company.

company	route 1	route 2	route 3	route 4	route 5
A	2500	3000	1500	–	6000
B	–	4000	–	4000	7000
C	3000	2000	2000	3500	–

The problem is to decide which company should service each line so that the overall expense of the city is minimized. Formulate the problem as a network optimization problem and solve it through an appropriate algorithm.

38

In a shop, three machining centers must produce 7 batches of mechanical parts. Batches can be of three types, namely rods, pistons or carters. Each machining center requires exactly 1 hour to produce a batch of parts, but a center can only produce the part types indicated in the table.

center	part types
A	rods, pistons, carters
B	rods, pistons
C	carters

It's 8 am. Center A is only available until 10 am, the other two until 11 am. Moreover, each batch should be completed within a certain due time.

batch	type	due time
1	rods	9
2	rods	11
3	pistons	9
4	pistons	11
5	carters	10
6	carters	11
7	carters	10

The problem is to determine whether is it possible to complete all batches within their respective due times. Formulate the problem as a network optimization problem.

39

In the otherworldly Nangijala there exist several currencies, corresponding to the nodes of the oriented graph in Figure 9. The weight of arc (i, j) indicates how many units of currency j one can buy with a unit of i. So, for instance, a tolar buys 32 shields. Jonatan has one gold coin, and wants to buy as many reals as possible. How can he do? Formulate the problem as a network optimization problem and solve it through a suitable algorithm.

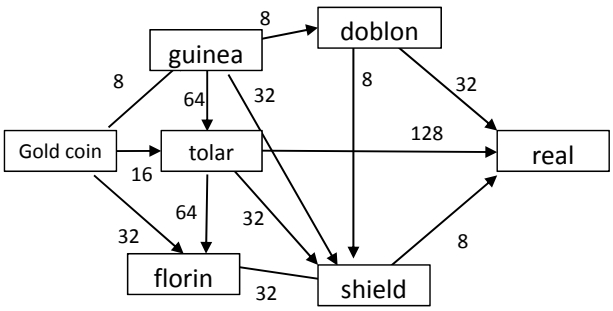

Figure 8: Graph for Problem 39.

40

It is midday. In the Fiumicino airport of Rome, three operators (Ann, Bill and Chuck) must prepare the flight plans for some outgoing flights. An operator takes exactly 1 hour to prepare a flight plan, but while Ann can prepare any flight, Bill can only prepare plans of domestic or medium-haul flights, and Chuck only long-haul. Ann terminates her shift at 2pm, the other two at 3pm. Each flight plan must be ready by a certain due time. The problem is to determine whether is it possible to prepare all the plans complying with the due times. Formulate the problem as a network optimization problem.

flight	type	due time
Rome-Milan	domestic	1pm
Rome-Palermo	domestic	3pm
Rome-Helsinki	medium-haul	1pm
Rome-Berlin	medium-haul	3pm
Rome-Tokyo	long-haul	2pm
Rome-New York	long-haul	3pm
Rome-Vaduz	medium-haul	2pm
Rome-Chittagong	long-haul	1pm

41

In the research lab of a car manufacturing company, a set of tests have to be performed. Each test has to be executed in a specific time interval, and requires utilizing an engine for a certain amount of KEs (Kilometer-Equivalents). Each test, if performed, brings a revenue of w_i euros.

test	start day	end day	KEs	w_i
A	1	2	20000	2300
B	2	3	20000	1800
C	2	4	30000	4500
D	3	4	20000	3200
E	4	4	10000	1500
F	4	5	30000	3300
G	5	6	20000	3200
H	5	7	30000	4000
I	6	6	10000	1600
J	6	7	20000	2700
K	7	8	20000	3800

The lab has an engine having residual life of 50000 KEs. Obviously, only one test at a time can be performed. The problem consists in finding the set of tests to be performed

such that the revenue is maximized. Formulate the problem as network optimization (solution is not required).

42

The regional government must allocate construction works of a new highway connecting Beagletown to Norsewood. Four firms are interested in the process (1, 2, 3, 4). The highway is divided into nine consecutive motorway sections. Figure 1 illustrates the sections and their corresponding construction cost (in millions of €). The government wants to balance as much as possible the overall expenditure (160 millions) among the four firms. More precisely, denoting by W_j the amount paid to firm j, the governments want to allocate the sections among the firms so to minimize the sum of the deviations (in absolute value) between each W_j and the average value $160/4 = 40$:

$$\sum_{j=1}^{4} |W_j - 40|$$

Considering that:

- a firm can build only *consecutive* sections
- a firm cannot build more than 3 sections,

formulate the problem (WITHOUT solving it) as network optimization.

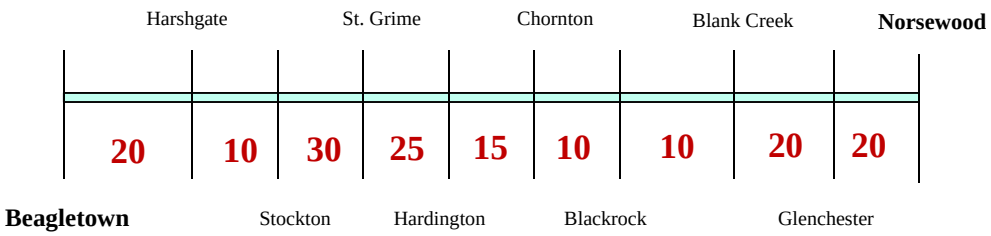

Figure 9: Highway outline for Problem 1.