Set membership localization and map building for mobile robots

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Summary. Autonomous navigation of mobile robots requires to continuously estimate the vehicle position and orientation in a given reference frame (localization problem). When moving in unknown environments, the more challenging problem of building a map, while at the same time localizing within it, must be faced (simultaneous localization and map building, SLAM). By adopting a landmark-based description of the environment, both tasks can be cast as a state estimation problem for an uncertain dynamic system, based on noisy measurements.

Under the assumption that both process disturbances and measurement errors are unknown but bounded (UBB), the estimation process can be carried out in terms of feasible sets. This work presents a review of efficient set membership localization and mapping techniques, for different kinds of available measurements and different classes of approximating regions. The proposed estimation algorithms are able to provide guaranteed set-valued estimates of the robot configuration as well as of the landmark locations. The choice of the structure of the approximating regions allows to achieve the desired trade-off between computational complexity and estimation accuracy. Moreover, the feasibility property of the computed estimates can be exploited to solve the measurement-to-feature matching problem, thus allowing to deal with indistinguishable landmarks. An extension of the SLAM algorithm to the case of a team of cooperating robots is also presented, under the additional hypotheses of distinguishable features and absolute orientation measurements. The proposed techniques are validated through extensive numerical simulations and experimental tests, performed in a laboratory setup.

1 Introduction

Self-localization of a mobile robot is a fundamental issue for achieving long-range autonomy. Almost all the tasks an autonomous agent is asked to perform, require the knowledge of the vehicle position within a global reference frame. The problem has been deeply studied in the last decades, and several solutions have been proposed, providing the estimates of the robot position, once a map of its surroundings is available. However, in real-world applications an autonomous agent is often called to face more challenging situations, where the operating environment is only partially known (uncertain map) or even completely unknown. In all these cases (e.g., exploration tasks or missions in hostile environments) a mobile robot must build a map of the environment it is navigating in, and simultaneously localize itself within the map. Consequently, in recent years, a great effort has been devoted to the development of efficient solutions to the Simultaneous Localization And Map building (SLAM) problem.

Localization and SLAM problems can be cast as state estimation problems for an uncertain dynamic system. Depending on the assumptions on the uncertainty, the estimation problem can be tackled in different ways. When a statistical description of the disturbances is adopted, standard solutions are provided by the Extended Kalman Filter (EKF) [26, 9, 31, 16, 34] or other probabilistic techniques [11, 33, 29]. All the methods based on the EKF generally model the uncertainty affecting the vehicle dynamics and the measurement process as zero-mean, white Gaussian noise. Unfortunately, real-world uncertainties seldom satisfy these hypotheses. Moreover, even when these assumptions are fulfilled, the Kalman filter is guaranteed to converge, but the EKF is not. Special care must be taken when neglecting correlated noise or systematic errors. In these cases, the Kalman filter estimates tend to be overoptimistic [22, 7]. These considerations motivated the adoption of alternative data fusion techniques. One of the most popular is set-theoretic estimators whose main feature is that the estimation uncertainty is represented by bounded sets in the state space.

One of the first papers in which the set-theoretic paradigm has been applied to robotic localization is [32], in which fusion of multiple sensor information with bounded uncertainty is used to construct a geometric representation of features in the environment. A set-based localization algorithm using images from a stereo vision system has been proposed in [3]. To cope with non-white, non-Gaussian noise, a set-theoretic approach to the problem of tracking a mobile robot, based on angular measurements, has been introduced in [20]. Under the hypothesis of bounded errors, information fusion is obtained via set intersections. This work has been later extended to a mixed stochastic/set-theoretic framework [19], in which a probabilistic uncertainty description is associated to ellipsoidal approximations of the admissible robot poses. In [30] a localization method is proposed, based on goniometrical observations of indistinguishable landmarks, taken by a panoramic camera. The robot evolution field is iteratively subdivided into rectangles containing at least one admissible landmark/measurement matching, until the desired estimation quality is achieved. In [23, 21] the problem of guaranteed robot localization is addressed via interval analysis, the optimal solution being computed via set inversion. Recently, a new set-theoretic localization methods, based on angular measurements, has been proposed in [6]. Recasting the original problem in a higher dimensional space, yields to accurate implicit polynomial descriptions of the admissible vehicle positions. As far as the SLAM problem is concerned, preliminary ideas on how to deal with bounded errors are given in [10, 15].

This paper presents a set-theoretic approach to localization and SLAM, which has been recently developed and successfully applied to several problems in mobile robotics [17, 15, 12, 13, 14, 8] The scenario considered in the paper consists of a mobile robot navigating in a *2D environment*, whose description is given in terms of point-wise features *(landmarks)*. The robot is supposed to be equipped with suitable sensors, providing only metric information related to the surrounding landmarks, which therefore turn out to be *indistinguishable*. The disturbances affecting both the robot dynamic model and the measurement process are supposed to be *unknown but bounded* (UBB), while no statistical assumptions concerning the nature of the errors are made. As a consequence, the solutions of the estimation problems related to localization and SLAM are given in terms of *feasible uncertainty regions*, i.e. sets containing the robot pose and/or the landmark positions. The basic reference theory for the technical development of the paper relies in the recently developed set membership estimation theory (see, e.g, [28, 27]). Though most of the theory is developed for linear estimation, the specific structure of the nonlinear localization problem can be exploited to get efficient solutions, based on recursive approximations of the uncertainty regions.

The Chapter is organized as follows. In Sections 2 and 3, the localization and SLAM problems, respectively, are stated in a set-theoretic framework. In Section 4, the strategy leading to efficient approximate solutions of the above problems is summarized. In Section 5, it is shown how to exploit the set-valued estimates in order to tackle the matching problem in presence of indistinguishable features. In Section 6, the extension of the SLAM algorithm to the case of a team of cooperating robots is outlined. In Section 7, a set-theoretic path planning algorithm, for designing minimum uncertainty trajectories, is presented. In Section 8, results of simulated and real-world experiments are discussed. Finally, some conclusions are drawn in Section 9.

2 Mobile robot localization

Let us consider a robot navigating in a 2D environment, whose *pose* (position and orientation) at time k is denoted by

$$p(k) = [x(k)\ y(k)\ \theta(k)]' \in \mathcal{Q}$$

with $\mathcal{Q} \triangleq \mathbb{R}^2 \times [-\pi\ \pi]$ being the set of all possible robot configurations. The coordinates $(x(k),\ y(k))$ represent the position of the vehicle, while its heading, w.r.t. the positive x-axis, is given by $\theta(k)$. Under the hypothesis of slow robot dynamics, a simple linear model can be adopted to describe the time evolution of the robot pose

$$p(k+1) = p(k) + u(k) + w(k), \tag{1}$$

where $u(k) \in \mathbb{R}^3$ denotes the measurements of robot displacements, coming from the odometric sensors and $w(k) \in \mathbb{R}^3$ models the error affecting such measurements.

As far as the localization problem is concerned, it is assumed that an accurate map of the environment is available, in terms of pointwise static landmarks, whose positions are *known*. Let

$$l_i = [x_{l_i} \; y_{l_i}]', \quad i = 1, \dots, n$$

denote the coordinates of the i-th landmark. The robot is supposed to be equipped with exteroceptive sensors, providing measurements related to each landmark. Depending on the sensory system, this information may consist in the distance of the vehicle from a landmark (provided by proximity sensors, such as sonars), or in the *visual angle* under which a landmark is seen (i.e. the angle between robot heading and the direction of the landmark, provided by panoramic cameras), or in both (provided by laser range finders or stereocams). In general, the measurement equations are nonlinear functions of the current robot pose $p(k)$ and of the coordinates of the sensed landmark. Since the latter are known, the sensor readings at time k can be described by equations of the form

$$c_i(k) = \mu_i(p(k)) + v_i(k), \quad i = 1, \dots, n \tag{2}$$

where $\mu_i(p(k))$ models the i-th measurement process and $v_i(k)$ represents the noise affecting that measurement. When a deterministic description of the uncertainty is adopted, no statistical hypothesis is made on the nature of the errors, the only assumption being that the disturbances are *unknown but bounded* (UBB):

$$|w_i(k)| \leq \epsilon_i^w(k), \;\; i = 1, 2, 3 \tag{3}$$

$$|v_i(k)| \leq \epsilon^{v_i}(k), \;\; i = 1, \dots, n \tag{4}$$

where $\epsilon_i^w(k)$ and $\epsilon^{v_i}(k)$ are known positive scalars. [1] The assumptions (3)-(4) naturally lead to a formulation of the localization problem in a set-theoretic framework. As a matter of fact, from equation (4) it is possible to define, for each measurement $c_i(k)$, a set

$$\mathcal{M}_i(k) = \{p(k) : \; |c_i(k) - \mu_i(p(k))| \leq \epsilon^{v_i}\} \tag{5}$$

representing all the vehicle poses $p(k)$ compatible with the sensor reading $c_i(k)$ and the UBB hypothesis of bounded error (4). If at time k the robot performs m different measurements, the admissible robot poses will be those belonging to the *measurement set*

[1] From now on, for notational convenience, the time dependency of the error bounds will be omitted.

$$\mathcal{M}(k) = \bigcap_{i=1}^{m} \mathcal{M}_i(k). \tag{6}$$

The set $\mathcal{M}(k)$ contains all the robot poses compatible with the all the measurements gathered at time k and with the bounds (4). If the hypothesis (4) of bounded errors are correct, the measurement set $\mathcal{M}(k)$ is not empty. Conversely, an empty intersection in (6) implies that at least one of the constraints (4) is violated.

The shape of each set $\mathcal{M}_i(k)$ depends on the nonlinear functions $\mu_i(p(k))$ modeling each measurement process. Hence, $\mathcal{M}_i(k)$ is, in general, a non convex set, bounded by nonlinear curves.

Since also process disturbances are supposed to be UBB, it is possible to introduce the notion of *feasible pose set* $P(k|k)$, defined as the set of all robot poses at time k compatible with all the information collected up to that time. Now, the set membership dynamic localization problem can be stated as follows.

SM Localization Problem. *Let* $P(0) \subset \mathcal{Q}$ *be a set containing the initial pose* $p(0)$*. Given the dynamic model* (1) *and the measurement equation* (2)*, find at each time* $k = 1, 2, \ldots,$ *the* feasible pose set $P(k|k) \subset \mathcal{Q}$ *containing all vehicle poses* $p(k)$ *that are compatible with the robot dynamics, the assumptions* (3)-(4) *on the disturbances and the measurements collected up to time* k*.*

The exact solution to the SM localization problem is given by the following recursion, which stems out directly from the robot dynamics (1) and the UBB assumptions (3)-(4)

$$P(0|0) = P(0) \tag{7}$$

$$P(k+1|k) = P(k|k) + u(k) + \mathrm{Diag}[\epsilon^w]\mathcal{B}_\infty \tag{8}$$

$$P(k+1|k+1) = P(k+1|k) \cap \mathcal{M}(k+1) \tag{9}$$

where $\epsilon^w = [\epsilon_1^w \ \epsilon_2^w \ \epsilon_3^w]'$, $\mathrm{Diag}[v]$ denotes the diagonal matrix with vector v on the diagonal and $\mathcal{B}_\infty$ is the unit ball in the ∞-norm.

The update of the feasible pose set is carried out within a prediction-correction scheme, as in the Kalman filter, processing at each time step the current odometric and exteroceptive measurements. Equation (8) is often referred to as *time update*: during this step, information provided by the robot dynamic model and odometric sensors are used to update the feasible set. The set $\mathrm{Diag}[\epsilon^w]\mathcal{B}_\infty$ describes all the admissible unmodeled dynamics and odometric errors satisfying assumption (3); due to these disturbances, the size of the feasible pose set grows during the time update (see Figure 1.a).

Equation (9) is known as *measurement update* as it exploits all the exteroceptive measurements to reduce the total robot uncertainty. The main property of recursion (7)-(9) is to provide, at each time step, all the pose values that are compatible with the available information: the true state is guaranteed to belong to set $P(k|k)$, and the size of such set gives a measure of the uncertainty associated to the estimate (see Figure 1.b).

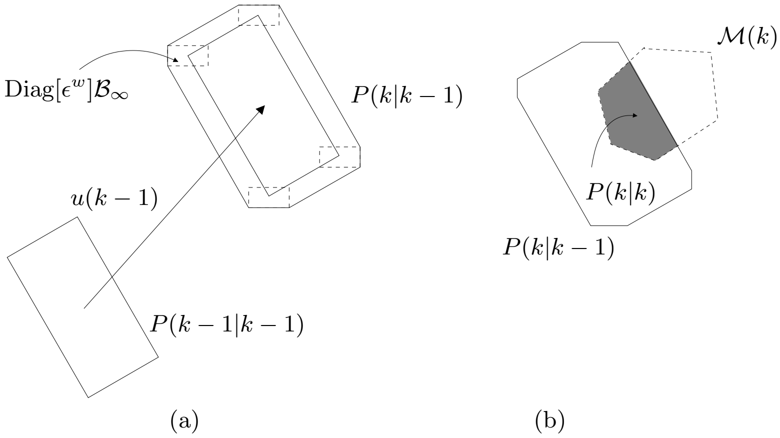

Fig. 1. Updating the feasible pose set: time update (a) and measurement update (b)

3 Simultaneous localization and map building

In several real world applications a map of the working space is not available. In those cases, the harder Simultaneous Localization And Map building (SLAM) problem has to be faced.

Let us consider the scenario outlined in Section 2. In order to state the SLAM problem as a state estimation problem, a model describing the time evolution of the landmark positions is needed. As long as only static landmarks are considered, their coordinates $l_i(k)$ satisfy the equations

$$l_i(k+1) = l_i(k). \tag{10}$$

Since both robot pose and landmark positions are unknown, the dimension of the state vector to be estimated can be very large, as it depends on the number of remarkable features present in the environment. Indeed, when n landmarks are considered, the state vector is given by

$$\xi(k) = [p'(k)\ l'_1(k) \dots l'_n(k)]' \in \mathbb{R}^{(3+2n)}. \tag{11}$$

From equations (1) and (10), the state update equation is

$$\xi(k+1) = \xi(k) + E_3 u(k) + E_3 w(k), \tag{12}$$

where $E_3 = [I_3\ 0 \dots 0]' \in \mathbb{R}^{(3+2n)\times 3}$. Concerning the exteroceptive measurements, equation (2) can be properly rewritten as

$$c_i(k) = \mu_i(p(k), l_i(k)) + v_i(k), \quad i = 1, \dots, n \tag{13}$$

to emphasize the dependence of the measurement process μ_i on the landmark position $l_i(k)$. Notice that in this case each equation (13) provides a nonlinear relations among some of the state variables (namely the robot pose and the i-th landmark coordinates).

Under the UBB assumptions (3)-(4), the definition of *measurement set* $\mathcal{M}(k)$ is still given by (6), provided that equation (5) is replaced by

$$\mathcal{M}_i(k) = \{\xi(k) : \ |c_i(k) - \mu_i(p(k), l_i(k))| \leq \epsilon^{v_i}\} . \tag{14}$$

Hence, the SLAM problem can be phrased in a set membership framework as follows.

SM SLAM Problem. *Let* $\Xi(0) \subset \mathbb{R}^{(3+2n)}$ *be a set containing the initial vehicle pose and landmark positions,* $\xi(0)$*. Given the dynamic model* (12) *and the measurement equations* (13)*, find at each time* $k = 1, 2, \ldots$ *the set* $\Xi(k|k)$ *of state vectors* $\xi(k)$ *which are compatible with the robot dynamics, the assumptions* (3)-(4) *on the disturbances and the measurements collected up to time* k*.*

The solution to the above problem is still provided by the algorithm outlined in equations (7)-(9), where the robot feasible pose set $P(k)$ is replaced by the extended feasible state set $\Xi(k)$

$$\Xi(0|0) = \Xi(0) \tag{15}$$

$$\Xi(k+1|k) = \Xi(k|k) + E_3 u(k) + E_3 \text{Diag}[\epsilon^w]\mathcal{B}_\infty \tag{16}$$

$$\Xi(k+1|k+1) = \Xi(k+1|k) \cap \mathcal{M}(k+1) \tag{17}$$

The *initialization* step of the algorithm, i.e. the choice of the set $\Xi(0)$, depends on the specific problem tackled. If there is no available information on the initial position of any element of the problem (i.e. robot and landmarks, as it happens in the SLAM case), the initial estimate set $\Xi(0)$ should be chosen as $\mathbb{R}^{(3+2n)}$. Nonetheless, since all measurements are relative, in this case one is allowed to choose an arbitrary reference frame. Hence, without loss of generality, it is possible to set the origin of the reference frame in the initial position of the robot, choosing as x-axis the robot initial heading. On the other hand, when an uncertain map of the environment is a priori given, $\Xi(0)$ can be chosen accordingly to the available information.

4 Suboptimal solutions

The major drawback of the above localization and SLAM algorithms is their high computational burden. The main reason of this is intimately related to the nature of the measurement sets $\mathcal{M}(k)$, defined in equations (5), (6) and (14). As already mentioned, such a set is given by the intersection of nonlinear and nonconvex sets, so that its shape can become arbitrarily complex. During the measurement update steps (9) and (17), such complexity affects the feasible state set, making the overall recursion too computationally demanding for real-time applications. Moreover, the exact feasible set is not only expensive to compute, but also its explicit, analytical expression is hard to maintain, since as new measurements are processed, the complexity of its shape increases. Finally, as far as the SLAM problem is concerned, computational issues are worsened by the high dimension of the true state space. For these reasons, suboptimal solutions to the SM localization and SLAM problems, trading off computational complexity and estimation accuracy, are sought.

The guidelines inspiring the suboptimal strategy can be summarized as follows.

- The devised algorithm should provide a *recursive, efficient* solution, suitable for *on-line* implementation.
- The set-valued estimates should be *guaranteed* to contain the actual robot pose and landmark locations.
- Simple, analytical expressions of the feasible sets are desired.

To meet the above requirements, outer approximations of the feasible sets, through simple structure regions, are adopted. Specifically, two approximations are introduced at different stages, leading to suboptimal solutions:

- decomposition of the state vector into subsets of state variables;
- outer approximations of the true feasible subsets through classes of simple regions.

The computation of the feasible sets in the entire state space, can be replaced by the evaluation of their projections on suitable subspaces, whose Cartesian product gives an outer approximation of the overall feasible sets. This policy allows to perform sums and intersections in lower dimensional subspaces, yielding a significant reduction of the computational burden. This proves to be especially useful in the SLAM context, where a high dimensional state vector is involved. A natural choice is to estimate separately the *feasible robot position set* Ξ_{xy}, the *feasible robot orientation set* Ξ_θ and, in the SLAM case, the *feasible landmark* position sets Ξ_{l_i}. With this strategy, sequential updates of only 1D and 2D sets are required.

The exact computation of the intersections required during the measurement update, although performed in low dimensional spaces, is still computationally demanding, due to the complex shape of the measurement sets. In order to improve the algorithm efficiency, as well as to come up with tractable analytical expressions for the set-valued estimates, the exact feasible subsets can be approximated through regions belonging to a class $\mathcal{R}$ of fixed and simple structure sets. In order to minimize the estimation uncertainty, the minimum volume region in the chosen class, containing the corresponding feasible set, is selected (see Figures 2-3).

Efficient algorithms for localization and SLAM, using boxes or parallelotopes as approximating regions, have been proposed in [17, 12, 14]. The main features of the devised techniques can be summarized as follows.

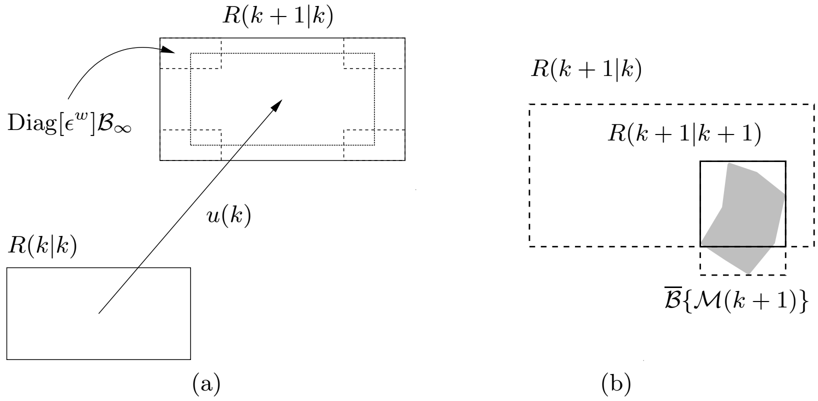

Fig. 2. Time update (a) and measurement update (b) using boxes. $\overline{\mathcal{B}}\{\mathcal{M}(k+1)\}$ denotes the minimum volume box containing the measurement set $\mathcal{M}(k+1)$

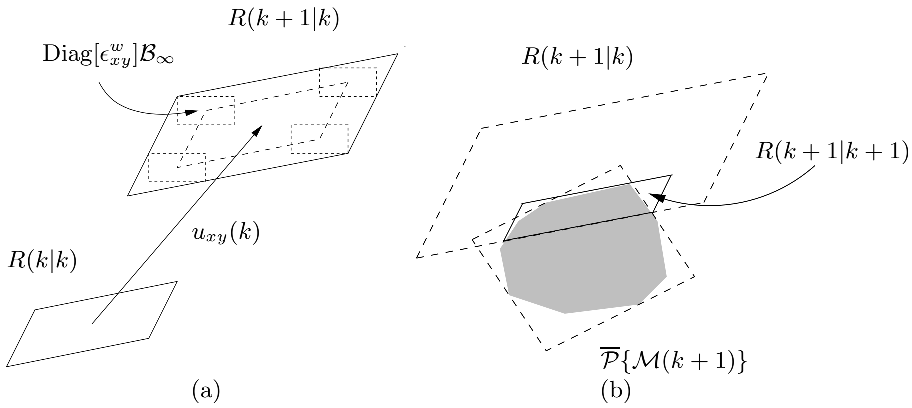

Fig. 3. Time update (a) and measurement update (b) using parallelotopes. $\overline{\mathcal{P}}\{\mathcal{M}(k+1)\}$ denotes the minimum volume parallelotope containing the measurement set $\mathcal{M}(k+1)$

- The estimation algorithms provide guaranteed set-valued estimates, in the sense that the actual state vector is guaranteed to belong to the computed uncertainty regions *(feasibility property).*
- No statistical assumptions on the nature of the errors are required, only an upper bound is assumed to be available.
- The suboptimal strategy adopted leads to fast algorithms suitable for on line implementation. Low computationally demanding estimates are obtained at the price of some conservativeness. This feature turns out to be especially useful when the SLAM problem has to be faced. In this case, the storage requirements of the proposed set membership techniques grows

linearly in the number of landmarks present in the environment, whereas its computational burden is linear in the number of features sensed at each time instant (as opposed to the quadratic complexity required by EKF-based algorithms).
- The desired trade off between computational effort and estimation accuracy can be achieved by a suitable choice of the class of approximating regions.
- Set approximations involved in the measurement update can be iteratively repeated, by processing several times the same measurements. This generally leads to more accurate approximations, at the price of a higher computational load. Moreover, in the SLAM problem this allows to implicitly account for the relationships between different state variables (see [14] for details).

The formulation of the localization and mapping problems presented so far implicitly assume that the robot is able to correctly associate each measurement to the corresponding sensed feature. For example, this happens if the observed landmarks are distinguishable. However, in real-world applications, especially if natural beacons are used as landmarks and sensors provide only metric information, landmarks are indistinguishable and consequently matching between measurements and landmarks is of paramount importance. In the next section, we will describe a data association algorithm exploiting the feasibility property of the set membership estimates [14].

5 Data association

At each time instant $k+1$, before the measurements are taken, the SM localization algorithm computes a region $R_{xy}(k+1|k)$ containing the prediction of the feasible robot positions according to the dynamic model (1) and the error bounds (3). Since measurement noise is supposed to be bounded, for each measurement c_i it is possible to define a set $\mathcal{M}_{l_i}(k+1)$ containing all the admissible i-th landmark locations (in a robot-centered reference frame) compatible with the noise bounds (4). Since the robot position is known up to an uncertainty region $R_{xy}(k+1|k)$, the set in which the i-th landmark is guaranteed to lie is

$$R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1).$$

As a consequence, any landmark position $l_j, j = 1, \ldots, n$ such that

$$l_j \in R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1), \tag{18}$$

can be associated to the i-th measurement. An example of such a set is depicted in Figure 4, when boxes are used as approximating regions and range and bearing measurements are available.

By considering each measurement, it is possible to build a matching matrix T, where all the admissible matchings measurement-landmark are listed. For

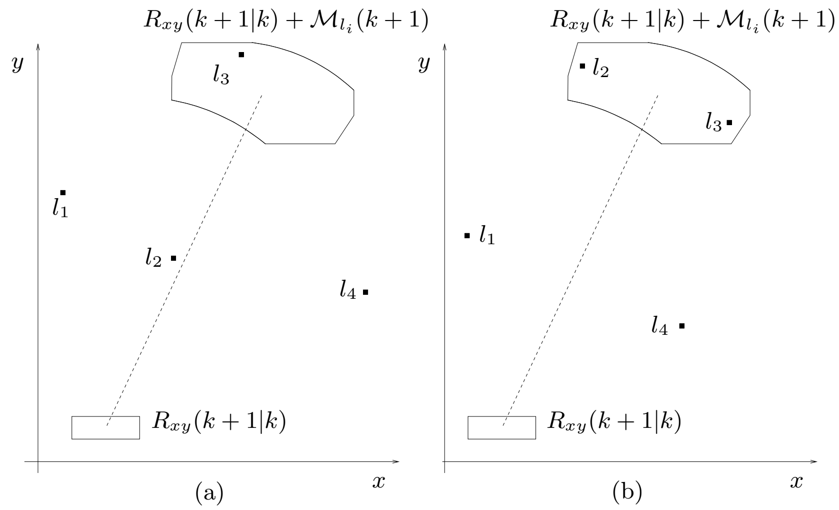

Fig. 4. Uncertainty sets can be employed to perform matching between measurements and features: (a) measurement i is not ambiguous, since only landmark l_3 lies in $R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1)$; (b) measurement i is ambiguous, since it can be associated both to landmark l_2 and to landmark l_3

instance, one can set $t_{ij} = 1$ if the j-th landmark lies inside $R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1)$, while $t_{ij} = 0$ otherwise. If on the i-th row of matrix T there is only one non zero entry, then the i-th measurement is not ambiguous. Should all the entries be null, than there is no landmark compatible with the measurement (consequently, the i-th measurement is a spurious one). Finally, a row with more than one nonzero entry, corresponds to an ambiguous measurement (see Figure 5a).

Even if some measurements are ambiguous, it is often possible to find a unique admissible solution to the matching problem (see Figure 5b). Indeed, the problem of determining the existence of a perfect matching is widely studied in the operation research field, and efficient algorithms are available to determine the solution of such problems [24].

The above procedure can be applied, with minor changes, also to the SLAM problem. As long as a map of the environment is not exactly known, in order to check whether a previously sensed landmark l_j can be accountable for the i-th measurement, the uncertainty of its current estimate must be suitably taken into account. Denoting by $R_{l_j}(k+1|k)$ the region approximating the feasible j-th landmark set at time $k+1$, this amounts to replace condition (18) by

$$R_{l_j}(k+1|k) \cap (R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1)) \neq \emptyset \qquad (19)$$

Accordingly, each landmark l_j satisfying (19) is eligible for the i-th measurement. The most significant difference with respect to the scenario of known landmarks, concerns the definition of spurious measurement. In a typ-

meas.\land.	a	b	c	d	e	f
1	0	1	0	0	1	1
2	1	0	1	0	0	0
3	0	0	0	1	0	1
4	1	0	0	0	1	0
5	1	1	0	0	0	0
6	0	0	0	0	1	0

(a)

meas.\land.	a	b	c	d	e	f
1	0	0	0	0	0	1
2	0	0	1	0	0	0
3	0	0	0	1	0	0
4	1	0	0	0	0	0
5	0	1	0	0	0	0
6	0	0	0	0	1	0

(b)

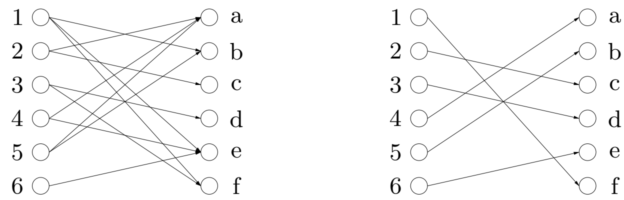

Fig. 5. Example of matching matrix T: (a) matrix with ambiguous measurements; (b) unique admissible matching

ical exploration task, where neither the locations nor the total number of remarkable features is a priori known, the landmarks to be mapped are usually incrementally discovered. In this case, an empty intersection between $R_{xy}(k+1|k) + \mathcal{M}_{l_i}(k+1)$ and every $R_{l_j}(k+1|k)$, may occur because:

- the i-th measurement is spurious, or
- the i-th measurement is related to a newly discovered landmark.

In the first case, one would simply discard the uninformative measurement. On the contrary, if a new feature is detected, the state vector should be properly augmented and the set-valued estimate of the new landmark should be initialized according to the set $R_{xy}(k+1|k)+\mathcal{M}_{l_i}(k+1)$. Unfortunately, there is no chance to certainly discriminate between these two options. Nonetheless, some heuristic techniques borrowed from statistical approach can be adopted, in order to avoid the introduction of spurious landmarks. The inclusion of a new feature into the state vector may be deferred until sufficient evidence of its presence is gathered, using a tentative list (see, e.g., [16]). A tentative landmark is initialized on receipt of a measurement and is then inserted into the state vector when a sufficiently high number of consecutive hits is reached. A possible alternative is to generate many data association hypotheses whenever a measurement is taken, and later discard all of them but one as more sensor data are collected (see, e.g., [5]).

6 Cooperative SLAM

In recent years, the employment of a team of cooperating agents to carry out complex tasks, has received more and more attention by the robotics research community (see, e.g., the special issues [2, 4, 1]). Fusion of information provided by different robots moving in the same area can indeed improve the exploration performance for several reasons. At each time instant the same feature can be perceived by more than one robot. If the robots share mapping information, this can lead to a more accurate, faster converging global map. Moreover, each robot plays the role of a moving landmark for all other agents, thus improving localization accuracy in poorly informative environments. For instance, if only one robot at a time is moving, the others can act as a landmark base in regions where it is difficult to extract reliable features [18].

The SM localization and SLAM algorithms can be naturally extended to the cooperative scenario. Let

$$p_j(k) = [x_j(k)\ y_j(k)\ \theta_j(k)]'$$

denote the pose of the j-th robot at time k. The setting hereafter considered consists of M agents, moving in an environment containing n unknown static landmarks $l_1, \ldots, l_n$. Then, the state vector to be estimated becomes

$$X(k) = [p_1'(k)\ \ldots\ p_M'(k)\ l_1'\ \ldots\ l_n']' \in \mathbb{R}^{3M+2n}. \tag{20}$$

Following the suboptimal approach illustrated in Section 4, the underlying idea is to decompose the approximation of the overall feasible set into $M+n$ approximations of 2D feasible subsets for the position of each feature in the environment, plus M interval approximations for the feasible orientation of each robot.

The main issue to be addressed is the fusion of the local maps, built by each agent, into a global one, when the relative starting positions of the agents are completely unknown. To this purpose, two additional assumptions are needed: (i) each feature is supposed to have a unique signature (distinguishable landmarks); (ii) absolute orientation measurements (such as the ones provided by compasses) are available. Nonetheless, according to the devised algorithm, map fusion is performed only once, at the beginning of the exploration in order to generate a common map, in a global reference frame. Afterwards, each agent is able to consistently update the global map even in presence of indistinguishable features and only relative bearing information.

In the single robot SLAM problem, since all measurements are relative, the initial position of the exploring agent can be fixed arbitrarily (in the robot centered reference frame) and all the environment features are then estimated with respect to that initial position. When operating with multiple agents, whose initial positions are not known, the global reference frame shared by all the robots must be chosen carefully. Assuming that absolute orientation measurements are available, it is possible to rotate the initial 2D uncertain maps onto an absolute orientation reference frame. By computing the initial 2D feasible position sets of each feature, the j-th agent is able to produce a self-centered initial map of the environment, within an absolute orientation reference system. This means that only relative translation among the different initial maps produced by the robots is unknown. Clearly, all the single-robot maps are a valid, suboptimal representation of the environment. The quality of the maps will generally vary from robot to robot. While it is possible to choose the "most accurate" map as a global one for all the robots, this choice does not use all the available information. Exploiting the feasibility property, it is possible to obtain a global refined map, by finding the minimum uncertainty description which satisfies all the constraints of each map. Moreover, when box-based approximations are adopted, the 2D map fusion can be decomposed into two 1D problems, whose solutions can be efficiently computed via linear programming algorithms. The interested reader is referred to [13], for a detailed description of SM cooperative SLAM.

7 Set membership approach to path planning

Motion planning is recognized to be a hard problem since long time. Nonetheless, as far as mobile robots (characterized by few degrees of freedom) are concerned, several solutions have been proposed, which proved to be effective in practice (see [25] for a comprehensive review). While uncertainty and disturbances are crucial issues in localization and map building, they have been often neglected in the path planning phase, i.e., the robot pose is usually assumed to be perfectly known.

In this section, we briefly show how the set-theoretic framework is well-suited to design minimum uncertainty paths. In particular, the problem of finding a trajectory minimizing the overall position uncertainty along the path, is tackled. This is motivated by applications in which it is crucial to localize the robot as precisely as possible along the whole path and not only in the final target, typical examples being the exploration of unknown environments or the accomplishments of several tasks requiring a prescribed precision along the travelled path. The appealing property of SM localization algorithms of delivering guaranteed set estimates, provides a simple way of quantifying the estimation uncertainty in terms of the size of the feasible pose sets.

Consider a robot with initial pose p_0 and whose target is to reach a pose p_T, after T moves. The objective is to plan a path $\mathcal{P} = [p(0), p(1), \ldots, p(T)]$, such that the average uncertainty associated to the path is minimized. This corresponds to tackle the following problem:

$$\begin{aligned}
&\min_{\mathcal{P}} \frac{1}{T} \sum_{k=1}^{T} V\left[\mathcal{M}(k)\right] \\
&\text{s.t.} \\
&|x(k+1) - x(k)| \leq b_x \;\; \forall k = 0 \ldots T-1 \\
&|y(k+1) - y(k)| \leq b_y \;\; \forall k = 0 \ldots T-1 \\
&p(0) = p_0 \; p(T) = p_T
\end{aligned} \tag{21}$$

where $V\left[\mathcal{M}(k)\right]$ is the volume of the feasible set $\mathcal{M}(k)$, and b_x, b_y are bounds on the maximum $x-$ and $y-$displacement for each move, respectively.

The solution of problem (21) presents several difficulties, since:

a) the sets $\mathcal{M}(k)$ depend on the actual realization of measurement noises;
b) the size of $\mathcal{M}(k)$ does not depend on the robot orientation $\theta(k)$ if the visibility range is unlimited and all landmarks can be seen from any robot pose; in the more realistic situation of limited field of view, $V\left[\mathcal{M}(k)\right]$ depends on the subset of features falling in the robot field of view.

Facing problem a) in a worst-case scenario, i.e. by maximizing $V\left[\mathcal{M}(k)\right]$ over all possible noise realizations, would lead to an intractable high-dimensional min-max optimization problem. A more viable approach is that of neglecting the measurement errors in the planning phase, which amounts to consider zero noise realization. Notice that this is generally an unfavorable case in set-theoretic estimation, where the maximum uncertainty reduction is usually achieved when the noise sequence takes values on the boundary as often as possible. In order to cope with problem b), the orientation interval can be suitably partitioned according to the subset of visible landmarks. By choosing the orientation subset for which the size of the feasible set $\mathcal{M}(k)$ is minimum, one can get rid of the dependence on θ in the cost function (21), thus reducing the number of optimization variables from $3T$ to $2T$. Moreover, the minimization problem can be further simplified if set membership localization techniques are used to approximate the feasible sets $\mathcal{M}(k)$. Notice that the proposed method is flexible enough to incorporate a priori knowledge on the robot working space. For instance, obstacles in the environment can be easily taken into account by combining artificial potential functions with the cost function in (21) (see Figure 6). Further details can be found in [8].

8 Experimental validation

The SM localization and SLAM algorithms outlined so far have been extensively tested in both simulated and real experiments.

Montecarlo simulations have shown that the SM localization algorithms are characterized by a good accuracy, fairly degrading with the increase of the measurement error bounds. Thanks to their flexibility, parallelotopic approximations usually delivers better performance, w.r.t. box-based ones. Extensive simulations have been performed also in different scenarios, showing

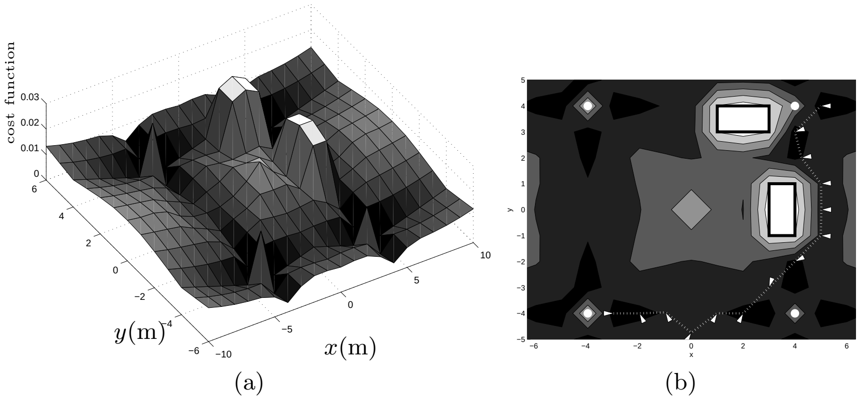

Fig. 6. Cost function (a) and computed path (b), for a scenario including two rectangular obstacles and four landmarks (white circles). Triangles represent the chosen robot poses along the optimal path

that SM algorithms are able to face the localization problem with different robot motion models and different exteroceptive sensors. Specifically, guaranteed estimates have been computed even in presence of correlated process disturbances or measurement noise. Moreover, the uncertainty affecting the set estimates turns out to rapidly approach its steady state value, especially when parallelotopic approximations are adopted (see [17, 12, 14]).

Similar simulation results have been obtained for the SM SLAM algorithm (see [15]). SM SLAM techniques have been compared to EKF-based algorithms, taking into account different noise models. Since a key performance index is the quality of the constructed map, the comparison is focused on the nominal error and the associated uncertainty of the final landmark position estimates. When measurement errors are modelled as white Gaussian noise, SM and EKF algorithms yields map estimates with comparable accuracy, whereas, in case of non-Gaussian, colored noise the SM approach performs usually better (see [14]). It is worth noticing that the set-theoretic framework requires a limited modelling effort (only upper bounds on the involved errors have to be set), while in the statistical context the noise covariance matrices have to be carefully tuned, in order to preserve EKF consistency. Moreover, a major advantage of the SM SLAM algorithm lies in its low computational burden and memory requirements. At each time step, the SM algorithm performs a number of operations proportional to the number of currently sensed landmark, whereas the EKF requires the propagation of the whole covariance matrix (a very expensive task when the number of landmarks increases). Such a feature is of paramount importance when exploring large areas, densely populated of landmarks.

Concerning the cooperative SLAM problem, simulations of the map fusion step have been run to evaluate the accuracy improvement of the global map

Fig. 7. Experimental setup: the mobile robots Nomad XR4000 (left) and Pioneer 3AT (right)

with respect to the maps built by each single robot. It turns out that the uncertainty reduction becomes more significant when the number M of robots (and consequently of available maps) increases. This happens because more maps will generally provide overall tighter constraints, in the map fusion step. In addition, for a fixed number of robots, improvements get less remarkable as the number n of non-sensing features increases, since adding non-sensing features to the environment increases the map uncertainty without providing additional means for uncertainty reduction. The SLAM algorithm has been simulated in a dynamic setting, in order to evaluate the accuracy improvement of robot localization and map estimation. The adoption of the cooperative scheme results in a faster convergence of landmark estimation accuracy to its steady-state value. In addition, each robot is able to precisely localize itself also at large distances from its starting point, thanks to the accurate map of initially faraway areas built by the other agents (see [13]).

The SM localization and SLAM techniques have been validated in a laboratory setup, featuring the mobile robots Nomad XR4000 and Pioneer 3AT (see Figure 7). These vehicles have different kinematics, the latter being a non-holonomic platform, while the former having a fully holonomic drive system. Both robots are equipped with a laser rangefinder and all the experiments involved indistinguishable, artificial landmarks, easily detectable from the laser scans. The proposed techniques are flexible enough to handle different vehicle motion models. The bounded error assumption turns out to be a reasonable description of the disturbances involved. The SM estimation techniques allow the robots to safely localize and navigate in spite of odometric error accumulation, and the data association algorithm proved to be effective when dealing with indistinguishable features. The results of the experimental tests are generally in good agreement with simulations and confirm the effectiveness of SM approach in real world applications (see [14]).

9 Conclusions

In this paper, a set-theoretic framework for addressing some relevant problems of autonomous navigation has been presented. Specifically, the attention has been mainly focused focused on localization and map building problems, for a single robot or a team of cooperating agents. Such problems can be cast as a state estimation problem for an uncertain dynamic system, based on non linear observations. Under the hypothesis of bounded model disturbances and measurement noise, the estimation process can be naturally stated in terms of feasible sets. Since real-time suitability represents an essential requirement, one of the main results is the design of efficient algorithms, trading off computational complexity and estimation accuracy. The extensive numerical simulations and experimental tests carried out, show that the set-theoretic approach represents a valuable alternative to statistical localization and map building methods, whenever the bounded error assumption holds.

A path planning algorithm for holonomic vehicles, which explicitly takes into account the localization uncertainty affecting the robot along its trajectory, has been devised. The proposed design procedure represents an attempt to consider in the planning stage the whole information available along the path. The next goal is the extension of the proposed planner when the more challenging simultaneous localization and map building problem has to be addressed. This should be intended as a first step toward the development of advanced exploration strategies, through the integrated solution of different tasks, like localization, map building and path planning. It is believed that set membership techniques provide a suitable framework for dealing with estimation uncertainty in such complex scenarios.

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