Complex systems and applications

Chiara Mocenni

Course on Complex Dynamic Systems

November 30, 2016
Plato and Pythagoras: everything in nature is a manifestation of the mathematical regularity.

Galileo: “The book of nature is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures” (1623).

Newton: Nature has its own laws and we can understand the regularities by means of mathematics” (1687).
“An intelligence that, at a given instant, could comprehend all the forces by which nature is animated ... would encompass in the same formula the movements of the greatest bodies of the universe and those of the lightest atoms. For such an intelligence nothing would be uncertain, and the future, like the past, would be open to its eyes.” (1825)
Henri Poincaré (1854-1912)

Poincaré created a new branch of mathematics: the qualitative theory of differential equations. He showed how it is possible to derive the most important information about the behavior of a family of solutions without having to solve the equation (since this may not always be possible).
Sensitivity to initial conditions

“A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance...

...even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, we should say that the phenomenon had been predicted.

But it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter... “ (HP, 1903).
Chaos Theory

- Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, an effect which is popularly referred to as the butterfly effect. Small differences in initial conditions yield widely diverging outcomes, rendering long-term prediction impossible in general. This happens even though these systems are deterministic, meaning that their future behavior is fully determined by their initial conditions, with no random elements involved. In other words, the deterministic nature of these systems does not make them predictable.
Characteristics of chaotic systems

- Deterministic dynamical nonlinear system
- Aperiodic behavior
- Sensitivity to initial conditions
- Stretching and folding
- Infinite repulsive cycles
From Chaos to Complexity

• Chaotic systems are examples of complex systems... why? ... and ... what are complex systems?

A complex system is a system composed of many components which interact with each other. In many cases it is useful to represent such a system as a network where the nodes represent the components and the links their interactions.

The principal signature of a complex system is the multiplicity of possible outcomes, endowing it with the capacity to choose, to explore and to adapt.
Emergent behavior

The emergence of traits encompassing the system as whole, that can in no way be reduced to the properties of the constituent parts. Emergent properties reflect the primordial role of interactions between parts.

Creation of self-organized states of a hierarchical and modular type, where order and coherence are ensured by a bottom-up mechanism rather than through a top-down design and control.

- Fluids under stress, e.g. Rayleigh-Benard cells in a fluid heated from below
- Open chemically reacting systems, e.g. bistability, oscillations, Turing patterns and wave fronts
- Communication and control networks in living matter, from the genetic to the organismic to the population level
Order and Disorder

Onset, within the same phenomenon, of large scale regularities and seemingly erratic evolutionary trends. Coexistence of order and disorder raises the issue of predictability of the future evolution of the system.

• The atmosphere: difficulty to issue reliable weather forecasts beyond a horizon of a few days

• Extreme geological and environmental phenomena: earthquakes and floods

• Human systems such as traders in stock markets influencing both each other and the market itself are also confronted to unexpected crises and collapses, despite the rationality supposed to prevail at the individual level.

• Fractals, deterministic chaos and its extreme form of fully developed turbulence show coexistence of order and disorder in time and space.
Applications of complex systems

Complex systems are at the origin of new techniques for artificial self-organizing and computational devices in such contexts as biotechnology, information science and robotics, where decentralized interactions of simple autonomous units lead to swarm intelligence and global structures complementary to those of conventional machines, able to respond and adapt with minimal outside direction, robust to damage and highly flexible.

Evolutionary principles stemming from complexity research also play a role in the understanding of the functioning of the immune system and the brain as well as in the development of artificial neural networks and related systems capable of performing pattern recognition, optimization, etc.
Applications of complex systems

Complex systems constitute a privileged interface between mathematical and physical sciences on the one side, and social and economic sciences on the other. It is natural to take advantage of the concepts and the techniques elaborated in the context of a physically-based complexity theory to tackle some of the problems arising in these disciplines from an interesting angle (analogy).

Market dynamics, management, transport, decision making bring the distinctive feature of being composed of individual elements with internal adaptation and response. This introduces a new level of complexity associated with the co-evolution of the components with each other and with their external environment.
Self-organization

Self-organization is a process in which pattern at the global level of a system emerges solely from numerous interactions among the lower-level components of the system. Moreover, the rules specifying interactions among the system's components are executed using only local information, without reference to the global pattern.
Self-organization

http://it.youtube.com/watch?v=sROKYelaWbo

https://www.youtube.com/watch?v=AYXktkfMnSI
Oscillations and self-organisation in the Millennium bridge

https://www.youtube.com/watch?v=eAXVa__XWZ8
Oscillations in a nonlinear electronic circuit

The Chua's circuit

\[
\begin{align*}
\frac{dx}{dt} &= \alpha[y - x - f(x)] \\
RC_2 \frac{dy}{dt} &= x - y + Rz \\
\frac{dz}{dt} &= -\beta y
\end{align*}
\]
Unstable oscillations in the Tacoma bridge

https://en.wikipedia.org/wiki/Tacoma_Narrows_Bridge_(1940)
Chaos plays several crucial roles; the system is designed and built so as to ensure its own steady and controlled source of "noise" (i.e., chaos), then chaotic behavior serves as the essential ground state for the neural perceptual apparatus (W. Freeman).
Chaos and Perception

Perceptual processing is not a passive process of reaction, in which whatever hits the receptors is registered inside the brain. Perception does not begin with causal impact on receptors; it begins within the organism with internally generated (self-organized) neural activity that lays the ground for processing of future receptor input.

Perception is a self-organized dynamic process of interchange inaugurated by the brain in which the brain fails to respond to irrelevant input, opens itself to the input it accepts, reorganizes itself, and then reaches out to change its input.

We suggest that the self-organized process that replaces environmental input with an internally generated, chaotic activity pattern is one that gives "biological meaning" to the stimulus.
The Turing patterns and bifurcation

The Chemical Basis of Morphogenesis is an article written by the English mathematician Alan Turing in 1952 describing the way in which non-uniformity (stripes, spots, spirals, etc.) may arise naturally out of a homogeneous, uniform state. The theory (reaction-diffusion theory of morphogenesis) has served as a basic model in theoretical biology and is seen by some as the very beginning of chaos theory. Alan Turing, 1952
Animal coats
Spatio-temporal chaos

Disordered behavior in time and space

Spontaneous formation of spazio-temporal patterns
The Belousov-Zabotinsky reactions are far from equilibrium and remain so for a significant length of time and evolve chaotically. They provide an interesting chemical model of nonequilibrium biological phenomena, and the mathematical models of the BZ reactions themselves are of theoretical interest and simulations.
Fractals and self-similarity

http://www.youtube.com/watch?v=XwWyTts06tU
https://www.youtube.com/watch?v=OjGaio87u3A
Complex networks

Complex systems are composed by numerous simple elements interconnected nonlinearly, giving rise to complex behavior, emergent behavior, self-organisation, evolution, adaptation due to rich dynamics. Differently from linear systems, the observed behaviour is different from what we may deduct from the behaviour of the elementary components of the system.

Interconnection is related to the idea of networks. Examples of networks are: internet, metabolic networks in a cell or organism, social networks, neural networks in the brain, networks of organisations, traffic, energy networks, communication networks, and so on.

Networks can be regular or disordered, or....
Regular, random and “small world” networks
The small world effect

A small-world network is a type of mathematical graph in which most nodes are not neighbors of one another, but the neighbors of any given node are likely to be neighbors of each other and most nodes can be reached from every other node by a small number of hops or steps.

In the context of a social network, this results in the small world phenomenon of strangers being linked by a short chain of steps. Many empirical graphs show the small-world effect, e.g., social networks, the underlying architecture of the Internet, wikis such as Wikipedia, and gene networks.
The vocabulary of complexity

EMERGENCE  SELF-ORGANIZATION
NONLINEAR DYNAMICS
NETWORKS  SYSTEMS THEORY
EVOLUTION AND ADAPTATION
GAME THEORY  PATTERN FORMATION
COLLECTIVE BEHAVIOUR
And to finish... the movie Jurassic Park

ENG
http://www.youtube.com/watch?v=n-mpifTiPV4

ITA
http://www.youtube.com/watch?v=iTlJR-kyBao
References

• Peitgen H.O., La bellezza dei frattali, (1987)

• Stewart I., Dio gioca a dadi? (1993)


• Gleick J., Caos. La nascita di una nuova scienza (1987)

• Vulpiani A. Determinismo e caos (1994)